

Due date: Thursday, May 8 **Deadline:** Thursday, May 15, 9:00a.m.

Amended May 8 and May 12

B means a problem in Blundell & Blundell's text; GT means a problem in Gould & Tobochnik.

1. B26.6 The first part is essentially the same as 26.5. Hint: For the last line, use eq. (11.26) and the reciprocity theorem. The full result is

$$C_p - C_v = \frac{R}{1 - \frac{2a(V-b)^2}{RTV^3}}$$

To get the result in the problem, you must assume $V \gg b$ and $VTR \gg a$. What is the physical meaning of each of these assumptions? Is each reasonable?

2. B28.3 Hints: a) Use the Clausius-Clapeyron equation; b) find the [conductive] heat flow from $\kappa (\Delta T/\Delta z)$; c) "eventually" implies steady state, so that the heat flow through a fractional thickness x meters of ice is the same as through $(1-x)$ meters of water.
- 3&4. B29.2 and B29.6 (B29.2 is too short and easy to count as a full problem, while B29.6 is longer than a usual problem.) Note that in eqs. (29.31) and (29.32), n_j should be $n_j!$, so that the denominators in both cases are $n_j! (g_j - n_j)!$. Furthermore, in eq. (29.35), there is this same error along with another in the term with parentheses: the denominator should be $n_j! (g_j - 1)!$. In going from eq. (29.35) to (29.36), you should assume that $g_j \gg 1$ so that you can replace $g_j - 1$ by g_j .

The expression you should maximize is

$$\frac{S}{k_B} + \alpha \left(N - \sum_j g_j \bar{n}_j \right) + \beta \left(E - \sum_j g_j \bar{n}_j E_j \right)$$

This is, I believe the source of using β for $1/k_B T$.

5. Consider a Fermi gas in 2 rather than 3 dimensions.

a) Find the Fermi energy in terms of N and A . *Added note: It is easier to do parts a) & c) together, then part b)!!*

b) Show that the average energy of the fermions is $\epsilon_F/2$.

c) Show that the density of states is independent of ϵ and write it in terms of ϵ_F and N .

d) One can show (but you do not have to) that $\mu(T) = k_B T \ln(e^{\epsilon_F/k_B T} - 1)$.

Find the limits of $\mu(T)$ as i) T goes to 0 and ii) when $k_B T \gg \epsilon_F$ and iii) show that they make sense.

e) One can also show (but you do not have to) that for low temperature $U/N - \epsilon_F/2 = (\pi^2/6)(k_B T)^2/\epsilon_F$.

Find the associated specific heat and compare to the result in 3D.

6. B30.5