Study Guide for Final Exam, Phys 404, Spring 2007

ver. 1.1

The final exam covers the entire course but emphasizes the material since the seond midterm: over half the questions will come from chaps. 6 and 7. You are allowed a 1-page cheat sheet.

Review homework problems

Boltzmann factor, partition func'n Z, probabilities $\mathcal{P}(s) = Z^{-1}\exp(-\beta E(s))$; $Z = \Sigma_s \exp(-\beta E(s))$; $\beta \equiv 1/k_B T$

Z for specific cases: paramagnet: $Z = 2\cosh \beta\mu B$; oscillator: $Z = (1 - e^{-\beta\epsilon})^{-1}$; ideal gas $Z_{tr} = (L/\lambda_Q)^3$

$U = N\bar{E} = -N\,\partial \ln Z/\partial\beta$ $\quad C_V = (\partial U/\partial T)_V$ $\quad S(T) = \int_0^T \frac{C_V(T')}{T'}dT'$

Application to paramagnetism, diatomic-molecule rotation, proof of equipartition theorem
Maxwell speed and velocity distributions in 3D, also 1D, 2D. Mean, mode, rms

$F \equiv U - TS = -k_B T \ln Z$; $S = -(\partial F/\partial T)_N$ $\quad p = -(\partial U/\partial V)_{S,N}$; $B = -V(\partial p/\partial V)_{T,N}$
$\mu = -T(\partial S/\partial N)_{U,V} = (\partial U/\partial N)_{S,V} = (\partial F/\partial N)_{T,V} = (\partial F/\partial N)_{T,V} = G/N|_{T,p}$
Partition function for composite systems: distinguishable vs. indistinguishable
Ideal gas $Z_{tr} = (L/\lambda_Q)^3$, $Z = (VZ_{int}/v_Q)^N/N!$, counting modes in box, quantum length $\lambda_Q = h/\sqrt{2\pi m k_B T}$

Gibbs factor, grand partition function $\mathcal{Z}$, probabilities $\mathcal{P}(s) = \mathcal{Z}^{-1}\exp[-\{E(s) - \mu N(s)\}/k_B T]$
Langmuir adsorption isotherm

Density of states $\mathcal{g}(\epsilon) \propto \epsilon^{(d-2)/2}$ for $\epsilon \propto n^2$ and how it relates to N, U, etc.; for $\epsilon \propto n$, $\mathcal{g}(\epsilon) \propto \epsilon^{(d-1)}$
$\epsilon_F = (\hbar^2/2m)(3\pi^2 N/V)^{2/3}$ (for d=3) $\quad \mathcal{g}(\epsilon)$ has units 1/energy

Fermions: Pauli exclusion principle, partition function, Fermi-Dirac dist. $\bar{n}_{FD}(\epsilon) = 1/[e^{\beta(\epsilon-\mu)}+1]$
Degenerate Fermi gas at T=0: definitions T_F, ϵ_F; quantities U, p, B
Sommerfeld expansion, meaning, usage to get lowest order (in T) contributions to shift of chemical potential μ from ϵ_F, increase in U, $C_V \propto T$

$$\int_{-\infty}^{\infty} H(\varepsilon)\bar{n}_{FD}(\varepsilon)d\varepsilon = \int_{-\infty}^{\mu} H(\varepsilon)d\varepsilon + \frac{\pi^2}{6}(k_B T)^2 H'(\mu) + O(k_B T)^4$$

Bosons: Planck distribution for simple harmonic oscillator, photons, mode counting (again)
Planck spectrum ($\mu = 0$) plotted vs ϵ or vs. λ; total power radiated (eq. 7.99) (emissivity) (area) σT^4
Wien displacement law; $\lambda_{max} T$ = const.
Debye theory of lattice vibrations of solids: deficiency of Einstein model at low T, phonons and their [acoustic] dependence on n; idea of n_{max} ; $C_V \propto T^3$
Bose-Einstein condensation: N_0 and N_0/N; role of chemical potential and of convergence of integral of $\mathcal{g}(\epsilon)\,\bar{n}_{BE}(\epsilon)$ $\quad \bar{n}_{BE}(\epsilon) = 1/[e^{\beta(\epsilon-\mu)}-1]$
$N_{exc} = (T/T_c)^{3/2}N$ and $N_0 = N - N_{exc}$ (for $T < T_c$); consequent U, C_V, p

Ising model: energy = $J\,\Sigma_{\langle i,j\rangle} s_i s_j$ where $s_i = \pm 1$ (up or down) Mean field $k_B T_C = J \times$ (# nearest neighbors)

Important Figs.: 6.11, 6.12, 6.16, 7.6, 7.7, 7.9, 7.14, 7.16, 7.19, 7.26, 7.29, 7.31, 7.32, 7.33, 7.34, 7.37; and what they mean/how to use them