PROBLEM 5.5 (a) [5.19], with $\langle x \rangle_n = \frac{a}{2}$ (Problem 2.5) and $\langle x^2 \rangle_n = a^2\left(\frac{1}{3} - \frac{1}{2(n\pi)^2}\right)$ (Problem 2.5) $\Longrightarrow$

$$\langle (x_1 - x_2)^2 \rangle = a^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right) + a^2\left(\frac{1}{3} - \frac{1}{2m^2\pi^2}\right) - 2\cdot\frac{a}{2}\cdot\frac{a}{2} = \boxed{a^2\left\{\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2} + \frac{1}{m^2}\right)\right\}}.$$

(b) $\langle x\rangle_{mn} = \frac{2}{a}\int_0^a x \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}x\right)dx = \frac{1}{a}\int_0^a x\left[\cos\left(\frac{(m-n)\pi}{a}x\right) - \cos\left(\frac{(m+n)\pi}{a}x\right)\right]dx$

$= \frac{1}{a}\left\{\left(\frac{a}{(m-n)\pi}\right)^2\cos\left(\frac{(m-n)\pi}{a}x\right) + \left(\frac{ax}{(m-n)\pi}\right)\sin\left(\frac{(m-n)\pi}{a}x\right) - \left(\frac{a}{(m+n)\pi}\right)^2\cos\left(\frac{(m+n)\pi}{a}x\right) - \left(\frac{ax}{(m+n)\pi}\right)\sin\left(\frac{(m+n)\pi}{a}x\right)\right\}\Big|_0^a$

$= \frac{1}{a}\left\{\left(\frac{a}{(m-n)\pi}\right)^2\left(\cos((m-n)\pi) - 1\right) - \left(\frac{a}{(m+n)\pi}\right)^2\left(\cos((m+n)\pi) - 1\right)\right\}$. But $\cos((m\pm n)\pi) = (-1)^{m+n}$.

$= \frac{1}{a}\frac{a^2}{\pi^2}\left((-1)^{m+n} - 1\right)\left(\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right) = \begin{cases} \frac{a}{\pi^2}\frac{(-8mn)}{(m^2-n^2)^2}, & \text{if } m, n \text{ have opposite parity} \\ 0, & \text{if } m, n \text{ have same parity (i.e. both even or both odd)} \end{cases}$

So [5.21] $\Rightarrow$ $\langle (x_1-x_2)^2\rangle = \boxed{a^2\left[\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2}+\frac{1}{m^2}\right)\right] - \frac{128a^2m^2n^2}{\pi^4(m^2-n^2)^4}}$. (Last term present only when m, n have opposite parity).

(c) [5.21] $\Rightarrow$ $\langle (x_1-x_2)^2\rangle = \boxed{a^2\left[\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2}+\frac{1}{m^2}\right)\right] + \frac{128a^2m^2n^2}{\pi^4(m^2-n^2)^4}}$. (Last term present only when m, n have opposite parity.)

PROBLEM 5.6 (a) $\boxed{\psi(x_1,x_2,x_3) = \psi_a(x_1)\psi_b(x_2)\psi_c(x_3)}$.

(b) $\psi(x_1,x_2,x_3) = \frac{1}{\sqrt{6}}\left[\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) + \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) + \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) + \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) + \psi_c(x_1)\psi_a(x_2)\psi_b(x_3)\right]$

(c) $\psi(x_1,x_2,x_3) = \frac{1}{\sqrt{6}}\left[\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) - \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) - \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) + \psi_b(x_1)\psi_c(x_2)\psi_a(x_3) - \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) + \psi_c(x_1)\psi_a(x_2)\psi_b(x_3)\right]$

<u>**Problem 5.10**</u> (a) $\left\langle \frac{1}{|\mathbf{r}_1-\mathbf{r}_2|} \right\rangle = \left(\frac{8}{\pi a^3}\right)^2 \int \left\{ \int \frac{e^{-4(r_1+r_2)/a}}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}}\, d^3\mathbf{r}_2 \right\} d^3\mathbf{r}_1$

$$\{\} = 2\pi \int_0^\infty e^{-4(r_1+r_2)/a} \left\{ \underbrace{\int_0^\pi \frac{\sin\theta_2}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}}\, d\theta_2}_{} \right\} r_2^2\, dr_2$$

$$\frac{1}{r_1r_2}\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}\,\Big|_0^\pi = \frac{1}{r_1r_2}\left[\sqrt{r_1^2+r_2^2+2r_1r_2} - \sqrt{r_1^2+r_2^2-2r_1r_2}\right] = \frac{1}{r_1r_2}\left[(r_1+r_2)-|r_1-r_2|\right]$$

$$= \begin{cases} 2/r_1 & (r_2<r_1) \\ 2/r_2 & (r_2>r_1) \end{cases}$$

$$= 4\pi e^{-4r_1/a} \left\{ \frac{1}{r_1}\int_0^{r_1} r_2^2 e^{-4r_2/a}\, dr_2 + \int_{r_1}^\infty r_2 e^{-4r_2/a}\, dr_2 \right\}.$$

$$\frac{1}{r_1}\int_0^{r_1} r_2^2 e^{-4r_2/a}\, dr_2 = \frac{1}{r_1}\left[-\frac{a}{4} r_2^2 e^{-4r_2/a} + \frac{a}{2}\cdot\left(\frac{a}{4}\right)^2 e^{-4r_2/a}\left(-\frac{4r_2}{a}-1\right)\right]\Big|_0^{r_1}$$

$$= -\frac{a}{4r_1}\left[r_1^2 e^{-4r_1/a} + \frac{ar_1}{2} e^{-4r_1/a} + \frac{a^2}{8} e^{-4r_1/a} - \frac{a^2}{8}\right].$$

$$\int_{r_1}^\infty r_2 e^{-4r_2/a}\, dr_2 = \left(\frac{a}{4}\right)^2 e^{-4r_2/a}\left(-\frac{4r_2}{a}-1\right)\Big|_{r_1}^\infty = \frac{ar_1}{4} e^{-4r_1/a} + \frac{a^2}{16} e^{-4r_1/a}.$$

$$= 4\pi \left\{ \frac{a^3}{32r_1} e^{-4r_1/a} + \left[-\frac{ar_1}{4} - \frac{a^2}{8} - \frac{a^3}{32r_1} + \frac{ar_1}{4} + \frac{a^2}{16}\right] e^{-8r_1/a} \right\}$$

$$= \frac{\pi a^2}{8}\left\{ \frac{a}{r_1} e^{-4r_1/a} - \left(2+\frac{a}{r_1}\right) e^{-8r_1/a} \right\}.$$

$$\left\langle \frac{1}{|\mathbf{r}_1-\mathbf{r}_2|} \right\rangle = \frac{8}{\pi a^4}\cdot 4\pi \int_0^\infty \left[\frac{a}{r_1} e^{-4r_1/a} - \left(2+\frac{a}{r_1}\right) e^{-8r_1/a}\right] r_1^2\, dr_1$$

$$= \frac{32}{a^4}\left\{ a\int_0^\infty r_1 e^{-4r_1/a}\, dr_1 - 2\int_0^\infty r_1^2 e^{-8r_1/a}\, dr_1 - a\int_0^\infty r_1 e^{-8r_1/a}\, dr_1 \right\}$$

$$= \frac{32}{a^4}\left\{ a\cdot\left(\frac{a}{4}\right)^2 - 2\cdot 2\left(\frac{a}{8}\right)^3 - a\cdot\left(\frac{a}{8}\right)^2 \right\} = \frac{32}{a}\left(\frac{1}{16} - \frac{1}{128} - \frac{1}{64}\right) = \frac{1}{a}\left(2 - \frac{1}{4} - \frac{1}{2}\right)$$

$$= \boxed{5/4a}.$$

(b) $V_{ee} \cong \frac{e^2}{4\pi\epsilon_0}\left\langle \frac{1}{|r_1 - r_2|} \right\rangle = \boxed{\frac{5}{4}\,\frac{e^2}{4\pi\epsilon_0}\,\frac{1}{a}} = \frac{5}{4}\,\frac{m}{\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 = \frac{5}{2}(-E_1) = \frac{5}{2}(13.6\,eV) = \boxed{34\,eV}$.

$E_0 + V_{ee} = -109\,eV + 34\,eV = \boxed{-75\,eV}$, which is pretty close to the experimental value (−79eV).

PROBLEM 5.13 (a) $E_F = \frac{\hbar^2}{2m}(3\rho\pi^2)^{2/3}$. $\rho = \frac{N_e}{V} = \frac{N}{V} = \frac{\text{atoms}}{\text{mole}} \times \frac{\text{moles}}{\text{gm}} \times \frac{\text{gm}}{\text{volume}} = \frac{N_A}{M} \cdot d$,

where N_A = Avogadro's number (6.02×10^{23}), M = atomic mass = 63.5 gm/mole, d = density = 8.96 gm/cm^3.

$$\therefore \rho = \frac{(6.02 \times 10^{23})(8.96\ \text{gm/cm}^3)}{(63.5\ \text{gm})} = 8.49 \times 10^{22}/\text{cm}^3 = 8.49 \times 10^{28}/\text{m}^3.$$

$$E_F = \frac{(1.055 \times 10^{-34}\ \text{J}\cdot\text{s})(6.58 \times 10^{-16}\ \text{eV}\cdot\text{s})}{(2)(9.109 \times 10^{-31}\ \text{kg})}(3\pi^2\, 8.49 \times 10^{28}/\text{m}^3)^{2/3} = \boxed{7.04\ \text{eV}}.$$

(b) $7.04\ \text{eV} = \frac{1}{2}(.511 \times 10^6\ \text{eV}/c^2)v^2 \Rightarrow \frac{v^2}{c^2} = \frac{14.08}{.511 \times 10^6} = 2.76 \times 10^{-5} \Rightarrow \frac{v}{c} = 5.25 \times 10^{-3}$, so $\boxed{\text{nonrelativistic}}$.

$$v = (5.25 \times 10^{-3}) \times (3 \times 10^8) = \boxed{1.57 \times 10^6\ \text{m/s}}$$

(c) $T = \frac{7.04\,eV}{8.62\times10^{-5}\,eV/K} = \boxed{8.17\times10^{4}\,K}$.

(d) $P = \frac{(3\pi^2)^{2/3}\hbar^2}{5m}\rho^{5/3} = \frac{(3\pi^2)^{2/3}(1.055\times10^{-34})^2}{5(9.109\times10^{-31})}(8.49\times10^{28})^{5/3}\ N/m^2 = \boxed{3.84\times10^{10}\,N/m^2}$.

<u>PROBLEM 5.14</u> $P = \frac{(3\pi^2)^{2/3}\hbar^2}{5m}\left(\frac{Nq}{V}\right)^{5/3} = AV^{-5/3} \rightarrow B = -V\frac{dP}{dV} = -VA(-\tfrac{5}{3})V^{-\frac{5}{3}-1} = \frac{5}{3}AV^{-5/3} = \frac{5}{3}P$. ✓

$\therefore$ for copper, $B = \left(\frac{5}{3}\right)(3.84\times10^{10}\,N/m^2) = \boxed{6.4\times10^{10}\,N/m^2}$.