QUANTUM PHYSICS II

PROBLEM SET 6

due October 20, before class

A. Anharmonic oscillator in first order perturbation theory

Find the correction of order $\mathcal{O}(\lambda)$ to the ground state energy and eigenket for the anharmonic oscillator described by the hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 + \lambda\hat{x}^4. \tag{1}$$

Notice: we discussed this in class but I want you here to fill in the details.

B. Step in the infinite well

Consider a particle moving in 1D under the influence of the potential:

$$V(x) = \begin{cases} V_0, \text{if } 0 < x < L/2, \\ 0, \text{if } L/2 < x < L, \\ \infty, \text{otherwise.} \end{cases} \tag{2}$$

i) Assume V_0 is small fing the eigenvalues of the hamiltonian correct up to first order in V_0.

ii) Perturbation theory is valid as long as the shift in energy due to the perturbation is indeed small compared to the energy level itself. What is the condition on V_0 so perturbation theory is valid? Is it more valid for low lying states or for the highly excited ones?