QUANTUM PHYSICS II

PROBLEM SET 2
due September 22, before class

A. Spin rotated and measured

A spin-$1/2$ particle is initially in the state

$$
\psi = \frac{\psi_+ + 2i\psi_-}{\sqrt{5}},
$$

where $\psi_\pm$ are the spin up and down states (along the z-axis). By means of magnetic fields the spin of the particle is rotated around the z-axis by an angle of $\pi$. At this point the x-component of the spin is measured. What are the possible outcomes and with which probabilities?

B. A spinor pointing towards an arbitrary direction

The operator corresponding to the spin projection along an arbitrary direction in space (parametrized by the unit vector $n$) is given by $\hat{S}_n = \hat{S}_x n_x + \hat{S}_y n_y + \hat{S}_z n_z$. Let $n = \sin \theta \cos \phi \mathbf{e}_x + \sin \theta \sin \phi \mathbf{e}_y + \cos \theta \mathbf{e}_z$. Find the eigenvectors of $\hat{S}_n$ with eigenvalue $+\hbar/2$ as a function of the angles $\theta, \phi$. Hint: you may want to start with the known eigenvector of $\hat{S}_z$ and apply rotations to align it in the $\theta, \phi$ direction. If you do it this way, make sure you verify at the end that the ket you obtained is indeed an eigenvector of $\hat{S}_n$.

C. Spin acrobatics

An electron is at rest in an oscillating magnetic field

$$
B = B_0 \cos(\omega t) \mathbf{e}_z,
$$

where $B_0$ and $\omega$ are constants.

(a) Construct the hamiltonian matrix (in the basis $\psi_+, \psi_-$ we have been using) for this system.

(b) The electron starts out (at $t = 0$) in the spin-up state with respect to the x-axis. Determine the state of the electron spin at any subsequent time. Beware this is a time-dependent hamiltonian, so you cannot find $\psi(t)$ in the usual way from stationary states. Fortunately, in this case you can solve the time-dependent Schroedinger equation (and feel free to use computers).

(c) Find the probability of getting $-\hbar/2$ if you measure $\hat{S}_x$.

(d) What is the minimum field ($B_0$) required to force a complete flip in $\hat{S}_x$?