QUANTUM MECHANICS II

PROBLEM SET 10

due December 10, before class

I. MORE ELECTRON ACROBATICS

An electron is at rest at the origin in the presence of a magnetic field constant in direction but whose directions spins around:

$$\mathbf{B}(t) = B_0(\sin\alpha\cos(\omega t)\mathbf{e}_x + \sin\alpha\sin(\omega t)\mathbf{e}_y + \cos\alpha\mathbf{e}_z). \tag{1}$$

i) Construct the 2×2 Hamiltonian matrix.

ii) Assuming the spin is initially up, show that the spinor describing the electron at a later time t is given by

$$\chi(t) = \begin{pmatrix} \left(\cos(\lambda t/2) + i(\frac{\omega+\omega_1\cos\alpha}{\lambda})\sin(\lambda t/2)\right)e^{-i\omega t/2} \\ i\omega_1\frac{\sin\alpha}{\lambda}\sin(\lambda t/2)e^{i\omega t/2} \end{pmatrix}, \tag{2}$$

where

$$\omega_1 = -\frac{eB_0}{m},\ \lambda = \sqrt{\omega^2+\omega_1^2+2\omega\omega_1\cos\alpha}. \tag{3}$$

iii) Now treat the same problem in first order perturbation theory and compare the approximate solution to the exact one. State carefully the condition that needs to be satisfied for perturbation theory to be valid.