QUANTUM MECHANICS II

PROBLEM SET 9

due November 7th, before class

I. VARIATIONAL CALCULATION FOR THE ANHARMONIC OSCILLATOR

Use the variational ansatz

$$\psi_\alpha(x) \sim e^{-\alpha x^2} \tag{1}$$

to estimate the ground state energy of the enharmonic oscillator described by the hamiltonian

$$H = \frac{\hat{p}^2}{2m} + \lambda \hat{x}^4. \tag{2}$$

Notice that I'm setting $|omega = 0$ to simplify the problem. Try to repeat the "exact" calculation of the last week for the present case with $\omega = 0$ and compare it with the variational result you just got.

II. VARIATIONAL METHOD FOR EXCITED STATES

Prove the following generalization of the variational principle: the first excited state is the normalized state *orthogonal to the ground state* with the smallest value of (the expectation value of) the energy. Use this result to estimate the first excited state of the harmonic oscillator. It is up to you to make a good ansatz so different people will have different solutions to this problem. How does your estimate compared to the exact solution?

III. FEYNMAN-HELLMANN AND THE EXPECTATION VALUES OF $1/r$ AND $1/r^2$ IN THE HYDROGEN ATOM.

Solve problems 6.32 part a and 6.33 parts a and b of Griffths.