QUANTUM PHYSICS II

PROBLEM SET 7

due October 24, before class

A. Perturbation Theory for the Anharmonic oscillator

The anharmonic oscillator is described by the hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 + \lambda\hat{x}^4. \tag{1}$$

Its eigenvalues cannot be obtained analytically. However for small enough λ we can use perturbation theory taking the harmonic oscillator as the unperturbed hamiltonian. Compute the shift in the energy of the ground state up to order λ^2. Hint: the best way to compute the required matrix elements is to use raising and lowering operators so you don't have to compute any integral. But if you use the explicit form of the wave function you'll end up with integrals of the gaussian type that are not too bad.