C. initial state $= \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle) \longleftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$

in the usual basis

rotation operator $\longrightarrow \hat{R}(z,\pi) = \begin{pmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2}\end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i\end{pmatrix}$

$$\hat{R}(z,\pi)\begin{pmatrix}1\\i\end{pmatrix}\frac{1}{\sqrt{2}} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}-i\\-1\end{pmatrix}}_{\text{rotated stated}}$$

measure $\hat{S}_x$: eigenstates of $\hat{S}_x$ are $|x,+\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \longleftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

w/ eigenvalue $\hbar/2$

and

$|x,-\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \longleftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

w/ eigenvalue $-\hbar/2$

$$\langle x,+|\psi,\text{rotated}\rangle = \frac{1}{\sqrt{2}}(1\ 1)\frac{1}{\sqrt{2}}\begin{pmatrix}-i\\-1\end{pmatrix} = \frac{1}{\sqrt{2}^2}(-i-1)$$

$$\langle x,+|\psi,\text{rotated}\rangle = \frac{1}{\sqrt{2}}(1\ -1)\frac{1}{\sqrt{2}}\begin{pmatrix}-i\\-1\end{pmatrix} = \frac{1}{2}(-i+1)$$

$$\boxed{\begin{array}{l}\text{probability of measuring } \hbar/2: \left|\frac{1}{2}(-i-1)\right|^2 = 1/2 \\ \text{probability of measuring } -\hbar: \left|\frac{1}{2}(-i+1)\right|^2 = 1/2\end{array}}$$