QUANTUM PHYSICS II

PROBLEM SET 2

due September 12, before class

A. More bra & ket-ology

i) Find the eigenfuction of the exposition operator with eigenvalue y in the eigenbasis of the momentum operator

ii) Find the matrix elements of the position operator in the momentum basis.

iii) Compute

$$\langle x|\frac{\hat{p}^2}{2M}+\frac{M\omega^2}{2}\hat{x}^2|\psi\rangle \tag{1}$$

in terms of $\langle x|\psi\rangle=\psi(x)$.

B. Two-level system

A quantum system has a two-dimensional Hilbert space with an orthonormal basis $\{|A\rangle,|B\rangle\}$. The hamiltonian is given by

$$\hat{H}=E_0(|A\rangle\langle A|+|B\rangle\langle B|)+T(|A\rangle\langle B|+|B\rangle\langle A|), \tag{2}$$

where E_0 and T are positive real constants.

i) Compute $\hat{H}|\psi\rangle$ where $|\psi\rangle=(|A\rangle+|B\rangle)/\sqrt{2}$.

ii) A generic ket in this pace is given by $|\psi(t)\rangle=a(t)|A\rangle+b(t)|B\rangle$. Solve the time-dependent Schrödinger equation with the initial condition $|\psi(t=0)\rangle=|A\rangle$.

iii) If the energy is measured at time t, what are the probabilities for the different outcomes?

C. Measuring momentum in the ground state of a harmonic oscillator

i) What are the probabilities of finding momentum p_0 when measuring the momentum of a harmonic oscillator inits ground state (you can use the wave function of the harmonic oscillator energy eigenstates you computed in PHY 401, no need to derive them again) ?

ii) What is the wave function of the system after the value p_0 is found for the momentum?

D. Operator wizardry

The momentum operator is represented, in the position eigenbasis, as $\hat{p}=-i\hbar d/dx$.

i) Compute

$$e^{-i\frac{y}{\hbar}\hat{p}}f(x). \tag{3}$$

Hint: expand the exponential and remember the Taylor series expression.

ii) Show that $\Psi(x,t)=e^{-i\hat{H}t/\hbar}\Psi(x,0)$ satisfies the time-dependent Schrödinger equation (assuming $\hat{H}$ is time independent). It is said that $\hat{p}$ generates space translations (from item i) and $\hat{H}$ generates time translations (by item ii).