Lecture 35 Highlights

The unusual properties of superfluid helium can be semi-quantitatively understood in terms of the two-fluid model. At any temperature below T_λ the fluid is made up of two inter-penetrating but non-interacting fluids. The superfluid is the set of all ^{4}He atoms in the ground state, while the normal fluid is the set of all ^{4}He atoms in the excited states inside the box. Each fluid has its own density, and they are constrained to add up to the total density: $\rho_s + \rho_n = \rho$. The superfluid has no entropy, flows with zero viscosity, and has no turbulence (for slow enough flow rates). The normal fluid has viscosity and can carry entropy. An experiment that measures the moment of inertia of a set of parallel disks rotating in the fluid shows that the densities are temperature dependent. Above T_λ there is only normal fluid. Below T_λ the superfluid density monotonically increases from zero at the expense of the normal fluid density, and eventually all the fluid is converted to superfluid in the limit of zero temperature.

The superleak is made up of small pores that pin the normal fluid because of its viscosity. The superfluid can move through the small pores because of its zero viscosity and lack of a boundary layer. The oscillating disks placed in the fluid shows an enhanced moment of inertia because it entrains the normal fluid and drags it back and forth as it oscillates. The superfluid remains at rest as the disk oscillates.

A critical flow velocity of about 5 cm/s exists for superfluid ^{4}He. For flow velocities greater than this it is possible to create excitations in the superfluid because enough energy is available to begin promoting ^{4}He atoms out of the ground state, thus taking energy away from the flow. A finite energy gap in the excitation spectrum prevents this dissipation mechanism from acting at low flow rates.

The properties of superfluid ^{4}He can also be understood in terms of a macroscopic quantum wavefunction $\psi(\vec{r})$. This is a wavefunction whose magnitude and phase can vary with position, and whose magnitude squared is equal to the local superfluid density: $|\psi(\vec{r})|^2 = \rho_s(\vec{r})$. The wavefunction is complex and can be written as: $\psi(\vec{r}) = \sqrt{\rho_s(\vec{r})}e^{i\theta(\vec{r})}$. Next week in discussion we will start with this wavefunction and calculate the superfluid current density as $\vec{J}_s = \frac{\hbar}{m}\rho_s \vec{\nabla}\theta = \rho_s \vec{v}_s$, m is the mass of the ^{4}He atom and $\vec{v}_s$ is the superfluid velocity field . From this we see that a mass current of the superfluid arises from a gradient in the phase of the macroscopic quantum wavefunction , rather than a pressure gradient! We will also show next week that the circulation of the super-flow is quantized as: $\kappa = \oint \vec{v}_s \bullet d\vec{l} = \frac{nh}{m}$, where $n = 0, \pm 1, \pm 2, \ldots$ A rotating bucket of superfluid ^{4}He will acquire circulation in units of *h/m* called superfluid *vortices* (see the web site for pictures). Vortices are topological defects in the fluid flow that preserve the irrotational (or curl-free) nature of the flow: $\vec{\nabla} \times \vec{v}_s = 0$ s