PHYS 401 Homework---Due October 29

1. From the commutation relation $[\hat{x}, \hat{p}] = i\hbar$ and the definition of the creation and annihilation operators show that $[\hat{a}, \hat{a}^{+}] = i\hbar$.

2. Consider a harmonic oscillator in the nth energy eigenstate (where the ground state is n=0. Show that in such a state $\Delta x \Delta p = (n + \frac{1}{2})\hbar\omega_0$. Hint: You know how to compute the expectation value of using the creation an annihilation operators.

3. This problem concerns energy eigenstates of the harmonic oscillator.
   a. Show that
   $$\hat{x}\,\hat{p}|n\rangle = -\frac{i\hbar}{2}\left(\sqrt{n(n-1)}\,|n-2\rangle + \sqrt{(n+1)(n+2)}\,|n+2\rangle - |n\rangle\right)$$
   Hint: express the operators in terms of creation and annihilation operators.
   b. Show that $\hat{p}\hat{x}|n\rangle = -\frac{i\hbar}{2}\left(\sqrt{n(n-1)}\,|n-2\rangle + \sqrt{(n+1)(n+2)}\,|n+2\rangle + |n\rangle\right)$
   c. Show that this the previous two results are consistent with the commutation relation $[\hat{x}, \hat{p}] = i\hbar$

4. Consider a state of the harmonic that at time t=0 is given by
   $|\psi(t=0)\rangle = \frac{|n\rangle}{\sqrt{2}} + \frac{|n+1\rangle}{\sqrt{2}}$.
   a. Show that at future times $|\psi(t)\rangle = e^{-i(n+1/2)\omega_0 t}\left(\frac{|n\rangle}{\sqrt{2}} + e^{-i\omega_0 t}\frac{|n+1\rangle}{\sqrt{2}}\right)$
   b. Show that the time dependence of the expectation value of x in this state is given by $\langle x\rangle = \sqrt{\frac{(n+1)\hbar}{2m\omega_0}}\cos(\omega_0 t)$
   Hint: express x in terms of creation and annihilation operators.
   c. Show that the time dependence of the expectation value of p in this state is given by $\langle p\rangle = -\sqrt{\frac{(n+1)\hbar\omega_0}{2m}}\sin(\omega_0 t)$
   d. Show from the previous expressions that these expectation values “look like” classical mechanics in the sense that $\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m}$ and $\frac{d\langle p\rangle}{dt} = -K\langle x\rangle$

5. Show that $\langle n'|\hat{x}^{l}|n\rangle = 0$ unless $|n-n'| \le l$.