PHYS 401 Homework---Due September 24

1. Consider the one dimensional particle in a box problem studied in class (i.e with a potential given by $V(x)=\begin{cases} 0 \text{ for } 0 \le x \le a \\ \infty \text{ for } x<0 \text{ or } x>a \end{cases}$). Suppose that at time t=0 the wave function is $\psi(x,t=0)=A\sin^3\left(\frac{\pi x}{a}\right)$.
   a. Show that (up to an irrelevant phase) $A=\frac{4}{\sqrt{5a}}$.
   b. Use the orthonormality of the energy eigenfunctions to show that $\psi(x,t=0)=3\sqrt{\tfrac{1}{10}}\psi_1(x)-\sqrt{\tfrac{1}{10}}\psi_3(x)$ where $\psi_1(x)$ and $\psi_3(x)$ are the normalized solutions to the time independent Schrodinger equation

2. Use the results of 1. to write down the solution of the time dependent Schrodinger equation.

3. Use the results of 2. to:
   a. Find an explicit expression for the probability density $\rho(x,t)$.
   b. Find explicit expression for the probability current $J(x,t)$ defined in class.

4. Verify the local conservation of probability: $\frac{\partial \rho(x,t)}{\partial t}=-\frac{\partial J(x,t)}{\partial t}$ using the explicit expressions in 3.

5. Use the result of 3. to find $\langle x\rangle$ and $\langle x^2\rangle$ as a function of time for this system. If you do this correctly you should find $\langle x\rangle$ is time independent while $\langle x^2\rangle$ is time dependent. Explain how this is possible.

6. What is the probability that if you measured the energy that you would find it to be
   a. $\frac{\hbar^2\pi^2}{2ma^2}$
   b. $\frac{4\hbar^2\pi^2}{2ma^2}$
   c. $\frac{3\hbar^2\pi^2}{2ma^2}$
   d. $\frac{9\hbar^2\pi^2}{2ma^2}$