

PHYS 401 Homework---Due September 17

1. In class we discussed the Bohr atom. The derivation of the allowed energies was based on the electrostatic force, the quantization of angular momentum into integral units of \hbar and the assumption of circular orbits. Repeat this derivation supposing that the force holding a particle in its orbit is that of a three dimensional harmonic oscillator with a force given by $\vec{F} = -m\omega^2 \vec{r}$ with $\omega = \sqrt{k/m}$ where k is the spring constant and of the oscillator and m the mass of the orbiting particle. Show the allowable radii for the circular orbits and the allowable energies. How do these allowable energies compare with those of Planck?
2. Consider the following problem. A bead is constrained to move (without friction) along a wire. The wire is bent into a circular loop of radius R . Imposing the condition that the of angular momentum is quantized into integral units of \hbar , find the allowable energies.
3. Let us try to gain some insights into the quantization of L in problem 2. Suppose, following de Broglie that there is a wave function ψ which describes the system. This wave function can be described in terms of θ the angular variable describing motion around the ring. By analogy with de Broglie's description of linear motion one would expect the wave function for a state of good angular momentum to be $\psi(\theta) = e^{iL\theta/\hbar}$. In deriving this we simply replace the ordinary position, x , by the angular position, θ , and the ordinary momentum, p , by the angular momentum, L . In order that the wave function is well defined in the sense of being single-valued it must satisfy the condition $\psi(\theta + 2\pi) = \psi(\theta)$. Show that this condition implies that $L = n\hbar$ for integral n .
4. Find the differential operator \hat{L} which, when operated on the angular wave function in problem 3 gives us the value of L . Show that this operator satisfies $[\hat{\theta}, \hat{L}] = i\hbar$ (where $\hat{\theta}$ is the operator associated with the angle.)

Note that the treatment of angular momentum in problems 2-4 is, in fact much simpler than in full quantum mechanics. The simplicity comes about because the problem is constrained to motion confined to two-dimensions. Three dimensional quantum mechanics introduces a more complex structure.