

## PHYS 401 Homework---Due November 29

1. Consider scattering from a repulsive delta function potential of the form  $V(x) = g \delta(x)$ . One way to view this problem is as the limit of a finite potential such as a square well. Compute  $R$  and  $T$  as a function of energy for this problem by taking the analogous expressions for the finite square barrier and studying them in the limit  $a \rightarrow 0$  with  $g = 2Va$  (where  $V$  is the height of the barrier).

2. Consider the same problem as in 1. However, I want it solved by a completely different means: namely by integrating the Schrodinger equation from  $-\epsilon$  to  $\epsilon$  and looking at the limit  $\epsilon \rightarrow 0$  (as we did in class for the bound states).

3. Consider a periodic potential  $V(x) = V(x+a)$ . The energy eigenstates satisfy Bloch states theorem  $\psi(x) = e^{ikx}u(x)$  with  $u(x+d) = u(x)$ . Thus the energy eigenstates cannot be normalized. Never-the-less,

expectation values of a sort can be defined. 
$$\langle A \rangle = \frac{\int_{-d/2}^{d/2} dx \psi^* \hat{A} \psi}{\int_{-d/2}^{d/2} dx \psi^* \psi}.$$

Where  $\hat{A}$  is the explicit differential form of the operator.

- Show that if  $u(x)$  is real that  $\langle p \rangle = \hbar k$ .
- Given the definition for the expectation value is the average value for momentum you would expect if you made many measurements of systems in this state?
- Clearly  $\langle x^2 \rangle$  does not correspond to the the average value for  $x^2$  you would expect if you made many measurements as that value is infinite. What does  $\langle x^2 \rangle$  represent physically?