

PHYS 401 Homework---Due November 19

1. Consider an attractive square well between $-a$ and a with depth V . Suppose that the parameters are such that $2mVa^2 / \hbar^2 = (9\pi/4)^2$.
 - a. Using the properties of the transcendental equations derived in class determine how many bound states there are for the system. (Hint: consider the even and odd states separately).
 - b. Find the energies of these states in terms of the parameters of the problem and numerical values find by solving the transcendental equations. (You must solve these numerically)
 - c. For the lowest lying state what fraction of the wavefunction is in the classically forbidden region?
 - d. For the highest lying bound state what fraction of the wavefunction is in the classically forbidden region?
 - e. Why are the answers to c. and d. so different?

2. Consider the potential $V(x) = -\frac{V_o}{1 + (x/a)^2}$. Consider the case where $2mV_o a^2 / \hbar^2 \gg 1$ in which we expect many bound states. Thus the lowest lying states will set near the bottom where the potential is well approximated as a harmonic oscillator.
 - a. Show that near the minimum the potential is well approximated by $V(x) = -V_o + V_o(x/a)^2$.
 - b. Using the result in a. show that we can approximate the lowest lying states by $E_n = -V_o \left(1 - (2n+1) \sqrt{\frac{\hbar^2}{2ma^2V}} \right)$ $n = 0,1,2\dots$

3. Consider the problem discussed in 2. for the case $2mV_o a^2 / \hbar^2 = 400$.
 - a. Estimate the lowest three levels via a numerical solution of the Schrodinger equation. (Hint: You can adapt the Mathematica code on the web site)

b. How do these answers compare with the estimate in 2 b.

4. Consider the potential $V(x) = \frac{V_0}{1 + (x/a)^6}$ with $2mV_0a^2/\hbar^2 = 64$. Find the minimum value for the energy for which the transmission coefficient is unity and the second value for which the transmission coefficient is unity. (Hint: You can adapt the Mathematica code on the web site)