SUMMARY OF PRINCIPLES OF QUANTUM MECHANICS

- bra and ket notation:

$$|f\rangle \longleftrightarrow f(x)$$

$$\langle g| \longleftrightarrow \int_{-\infty}^{\infty} dx\, g^*(x) \underset{\text{slot for a function of } x}{\underbrace{\cdots}}$$

from these relations others can be deduced:

$$\langle g|f\rangle \longleftrightarrow \int_{-\infty}^{\infty} dx\, g^*(x) f(x)$$

$$|f\rangle\langle g| \longleftrightarrow f(x) \int_{-\infty}^{\infty} dy\, g^*(y) \ldots$$

$$\vdots$$

- The state of a system (at a given time) described by a ket

- observables correspond to hermitian operators

eigenvalues and eigenvectors of any hermitian operator $\hat{A}$ defined by $\hat{A}|n\rangle = a_n |n\rangle$

eigenvalue

eigenvector

— eigenvalues $a_1, a_2, \ldots$ are real

— eigenvectors $|1\rangle, |2\rangle, \ldots$ can be chosen to be orthonormal

$$\langle n|m\rangle = \delta_{nm}$$

if the indices n and m are continuous this becomes $\delta(n-m)$

— eigenvectors form a complete set = basis

$$\sum_n |n\rangle\langle n| = \mathbb{1}$$

identity operator

if the index n is continuous this becomes an integral

$\Downarrow$

any ket $|\psi\rangle$ can be written as a linear combination of the $|1\rangle, |2\rangle, \ldots$

$$|\psi\rangle = \mathbb{1}|\psi\rangle = \underbrace{(|1\rangle\langle 1| + |2\rangle\langle 2| + \cdots)}_{\sum_n |n\rangle\langle n|}|\psi\rangle = \underbrace{\langle 1|\psi\rangle}_{c_1}|1\rangle + \underbrace{\langle 2|\psi\rangle}_{c_2}|2\rangle + \cdots = \sum_n \langle n|\psi\rangle |n\rangle$$

- when the measurement of an observable A is made:
  - the possible outcomes are the eigenvalues of A: $a_1, a_2, \ldots$
  - the probability of getting the eigenvalue a_n is given by the (magnitude square of) the component of $|\psi\rangle$ in the direction of the corresponding eigenvector:

$$|\psi\rangle = \underbrace{\langle 1|\psi\rangle}_{\substack{\text{probability} \\ \text{of } a_1 \\ = |\langle 1|\psi\rangle|^2}} |1\rangle + \underbrace{\langle 2|\psi\rangle}_{\substack{\text{probability} \\ \text{of } a_2 \\ = |\langle 2|\psi\rangle|^2}} |2\rangle + \ldots$$

  - if the outcome of the measurement is a_n the state "collapses", just after the measurement, to $|n\rangle$:

$$|\psi\rangle \longrightarrow |n\rangle$$

- the time development of the system is determined by the Schrödinger equation:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \underbrace{\hat{H}}_{\text{hamiltonian}} |\psi(t)\rangle$$

if the eigenstates of $\hat{H}$ are known (let us call them $|n\rangle$), $\hat{H}|n\rangle = E_n |n\rangle$, it is easy find the time evolution for any initial state

$$|\psi(t=0)\rangle = \langle 1|\psi(t=0)\rangle |1\rangle + \langle 2|\psi(t=0)\rangle |2\rangle + \cdots$$

$$= \sum_n \underbrace{\langle n|\psi(t=0)\rangle}_{c_n} |n\rangle$$

c_n ← determined by initial condition

$$\Downarrow$$

$$\underbrace{|\psi(t>0)\rangle}_{\text{any future time}} = \sum_n \langle n|\psi(t=0)\rangle \, e^{-iE_n t/\hbar} |n\rangle$$