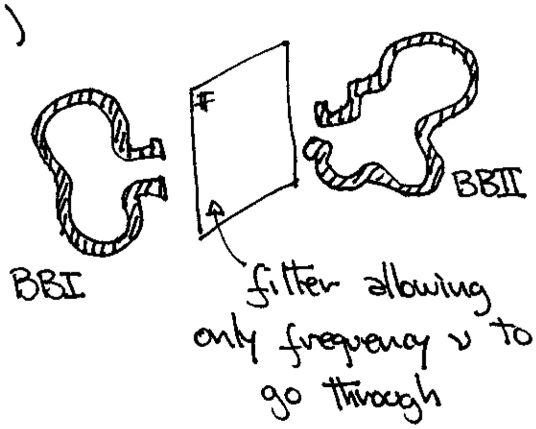
BLACKBODY RADIATION

- blackbody = body that absorbs all (electromagnetic) radiation falling on it

- many real objects (stars, a piece of metal) are reasonably approximated by blackbodies. The best way to build a blackbody is to build a cavity with a small hole (the hole acts as a blackbody)

- blackbody radiation depends only on T and ν (not on shape, material, ...)



in thermal equilibrium at temperature T :

$$\text{energy emitted by BBI at frequency } \nu \;=\; \text{energy absorbed by BBII at frequency } \nu \;=\; \text{energy emitted by BBII at frequency } \nu$$

because BBII is black

because BBII is in thermal equilibrium

- $$\underbrace{R_T(\nu)\, d\nu}_{\substack{\text{energy emitted by} \\ \text{unit of area and time} \\ \text{w/ frequencies between} \\ \nu \text{ and } \nu + d\nu}} = \frac{c}{4} \underbrace{u_T(\nu)}_{\substack{\text{energy density} \\ \text{inside the cavity of the} \\ \text{e.m. waves w/ frequencies} \\ \text{between } \nu \text{ and } \nu + d\nu}}$$

(NOT PROVED IN CLASS)

- Maxwell's eqs. show that e.m. waves inside of the cavity can be thought as harmonic oscillators, one for each allowed mode

$$E_x(x,y,z,t) = E_0 \cos k_x x \cos k_y y \cos k_z z \cos(\underbrace{c\sqrt{k_x^2+k_y^2+k_z^2}}_{= \frac{c}{L}\sqrt{n_x^2+n_y^2+n_z^2}})$$
$$E_y \cdots$$
$$\vdots$$

with $n_x, n_y, n_z = 0, 1, 2, \ldots$

In a box of volume V the number of allowed modes w/ frequencies between ν and $\nu + d\nu$ is (NOT PROVED IN CLASS)

$$N(\nu) = \frac{8\pi V \nu^2}{c^3}.$$

We can find the energy density $u_T(\nu)$ if we multiply $N(\nu)$ by the average energy of a harmonic oscillator w/ frequency ν at temperature T. Statistical mechanics says that this is given by (NOT PROVED IN CLASS)

$$\overline{E(\nu)} = \frac{\int_0^\infty dE \; e^{-E/kT} E}{\int_0^\infty dE \; e^{-E/kT}} = kT$$

- The classical expectation is then (due to Rayleigh and Jeans)

$$u_T(\nu) = \frac{8\pi \nu^2}{c^3} kT$$

This formula agrees w/ experiment at small ν but not at large ν where it, in fact, leads to an infinite total radiated energy $\int_0^\infty d\nu \, u_T(\nu) \to \infty$!

- Planck suggested that the energy levels of the harmonic oscillators were "quantized", that is, only energy levels equal to $E = nh\nu$ were allowed.

We have then

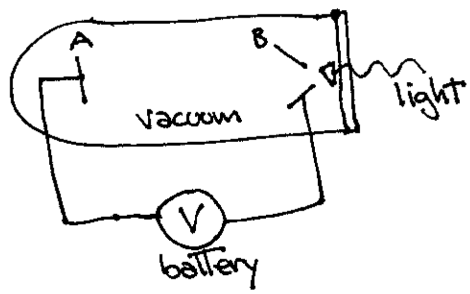
$$\overline{E(\nu)} = \frac{\sum_{n=0}^\infty e^{-nh\nu/kT} nh\nu}{\sum_{n=0}^\infty e^{-nh\nu/kT}} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

and

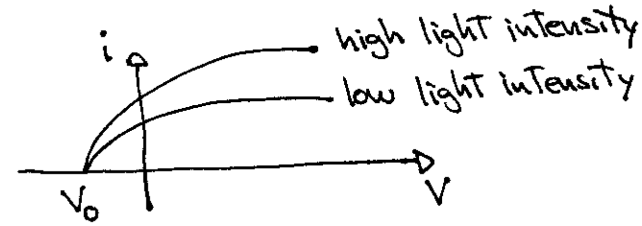
$$u_T(\nu) = \frac{8\pi \nu^3 h}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

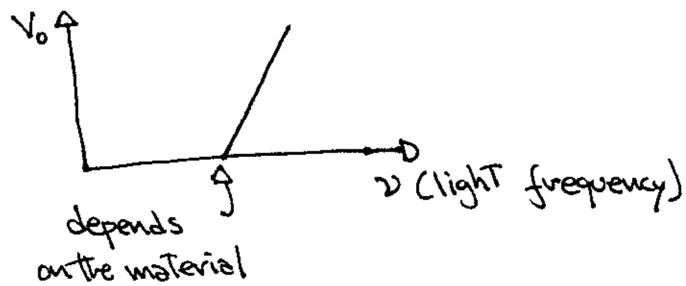
which, with $h = 6.6 \times 10^{-34}$ J.sec, agrees perfectly with experiment.

PHOTOELECTRIC EFFECT



Electrons leave A and reach B causing a current i when light shines on A. The basic empirical facts are:

1) 

2) 

3) There is no time lag between light and current

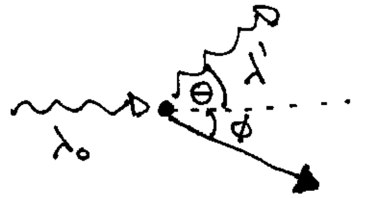
CLASSICAL EXPECTATION: The electric and magnetic fields of light move the charges in A. The higher the intensity of light, the higher the eletromagnetic fields, the larger the energy the charges acquire and the larger the potential barrier (V_0) they can overcome. Estimates of the time it would take for the elections to acquire energy enough ($\sim 1s$) indicates a time lag would be observed. No strong dependence on ν would be expected.

EINSTEIN'S "QUANTUM" EXPLANATION: Light is composed of "quanta" (photons) with energy $E = h\nu$. Higher light intensity means more photons, more electrons with increased energy and larger current. Electrons either absorb photons instantaneously or not at all, so there is no time lag. It takes a certain amount of energy w to extract an electron from the metal. For the least bound electron $w = w_0$ and

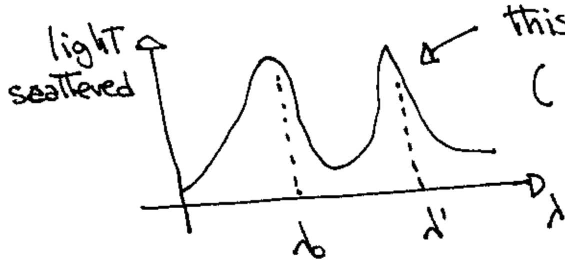
$$\underbrace{K_{max}}_{\text{max. kinetic energy}} = \underbrace{eV_0}_{\text{potential barrier}} = \underbrace{h\nu}_{\text{photon energy}} - w_0 \quad \Rightarrow \quad V_0 = \frac{h}{e}\nu - \frac{w_0}{e}$$

COMPTON EFFECT

Scattering of light by the (loosely bound) electrons of different materials



EMPIRICAL FACTS:

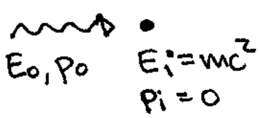


this peak depend on the angle θ (larger θ, larger $\lambda_0 - \lambda'$)

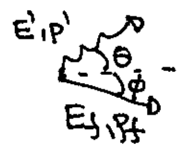
CLASSICAL EXPECTATION: The electromagnetic fields of the incoming wave makes the electrons oscillate with the same frequency (λ_0) and radiate with the frequency of the incoming wave ($\lambda' = \lambda_0$).

"QUANTUM" INTERPRETATION: Simple relativistic dynamics of the collision of a mass-less photon an a electron:

BEFORE



AFTER



$$\underbrace{h\nu_0}_{hc/\lambda_0} + mc^2 = \underbrace{h\nu'}_{hc/\lambda'} + \sqrt{p_f^2 c^2 + m^2 c^4} \qquad \text{(cons. of energy)}$$

$$\underbrace{p_0}_{h/\lambda_0} = \frac{h}{\lambda'}\cos\theta + p_f \cos\phi \qquad \text{(cons. of } p_x\text{)}$$

$$0 = \frac{h}{\lambda'}\sin\theta + p_f \sin\phi \qquad \text{(cons. of } p_y\text{)}$$

After a little algebra $\Rightarrow$

$$\boxed{\Delta\lambda = \lambda' - \lambda_0 = \underbrace{\frac{h}{mc}}_{\text{"Compton wavelength of the electron"}} (1 - \cos\theta)}$$