QUANTUM PHYSICS I

*PROBLEM SET 8*

*due December $8^{th}$ before class*

## A. Inescapable cylinder

The goal of this problem is to find some energy eigenfunction for a particle of mass $M$ inside a cylinder of length $L$ and radius $\rho$ with impenetrable walls (like the infinite square well but now in the shape of a cylinder).

i) Write the Schroedinger equation in polar coordinates and use separation of variables to split it in three separate ordinary differential equations

ii) Solve the equations for the $\theta$ and $z$ coordinates with the appropriate boundary conditions

iii) Look up (in the literature) the general solution for the radial equation. Hint: the solution is a linear combination of $J_m(\sqrt{2ME})$ and $Y_m(\sqrt{2ME})$, where $J_m$ and $Y_m$ are the so-called Bessel functions of $m^{th}$ order. How does $Y_m(x)$ behaves at small $x$? Is this acceptable physically?

iv) The first zeros of the $J_0(x)$ are at $x \approx 2.40483, 5.52008, 8.65373$ and $11.7915$ (this information is found on tables, Mathematica, Maple, wikipedia, ...). use that information to find the first four s-wave energy eigenstates.

## B. Spherical harmonics

i) Show that $Y_{lm}(\theta, \phi)$ is an eigenstate of $\hat{L}_z = -i\hbar\frac{d}{d\phi}$. What is the eigenvalue?

ii) Show that $Y_{1-1}(\theta, \phi)$ is an eigenstate of $\hat{L}^2$. What is the eigenvalue?

iii) Show that

$$e^{-\frac{i\varphi}{\hbar}\hat{L}_z}\psi(r, \theta, \phi) = \psi(r, \theta, \phi + \varphi). \tag{1}$$

Notice how this is similar to

$$e^{-\frac{iy}{\hbar}\hat{p}}\psi(x) = \psi(x + y) \tag{2}$$

proved in an earlier homework.