QUANTUM PHYSICS I

PROBLEM SET 6

due November 17^{th} *before class*

A. Eigenfunctions and eigenvalues of common operators

What are the eigenfunction and eigenvalues of the operators

i) $\hat{x}$

ii) $\hat{p}$

Repeat items i) and ii).

B. Eigenfunctions of kinetic energy

What are the eigenfunctions and eigenvalues of the kinetic operator $\hat{K} = \hat{p}^2/2m$. Show two degenerate eigenfunctions of the kinetic operator which are orthogonal to each other. Also, show two degenerate eigenfunctions that are NOT orthogonal.

C. Schroedinger equation in momentum space

Denote by $|k\rangle$ the momentum eigenfunction with eigenvalue $p = \hbar k$, that is

$$\hat{p}|k\rangle = \hbar k|k\rangle, \tag{1}$$

and by $|n\rangle$ the energy eigenfunction of the hamiltonian $\hat{H} = \hat{p}^2/2m + \hat{V}$ with eigenvalue E_n

$$\hat{H}|n\rangle = E_n|n\rangle. \tag{2}$$

Write the time independent Schroedinger equation in 2 in the basis of momentum eigenfunctions. You should obtain an equation for $\psi_n(k) = \langle k|n\rangle$ depending on the "matrix element" $\langle k|\hat{V}|k'\rangle$. Hint: the answer is

$$\psi_n(k)\left[\frac{\hbar^2 k^2}{2m} - E_n\right] + \int dk'\ \langle k|\hat{V}|k'\rangle\psi_n(k') = 0. \tag{3}$$

D. Probabilistic interpretation

A particle lives inside two impenetrable walls at $x = 0$ and $x = a$. Its wave function at time t is given by

$$\Psi(x,t) = \begin{cases} \frac{\sqrt{2}}{a}x, \text{for } 0 < x < a, \\ 0, \text{otherwise} \end{cases} . \tag{4}$$

i) If the energy is measured, what are the possible outcomes and with which probabilities?

ii) Suppose the energy measurement results in the value $E = 4\hbar^2\pi^2/(2ma^2)$. What is the expected (average) value of the position *immediately after* this measurement?