QUANTUM PHYSICS I

PROBLEM SET 5

due November 10^{th} *before class*

A. Muonic hydrogen

A muon is a particle identical to an electron except its mass is about 200 times larger. A muonic hydrogen is a bound state of a proton to a muon (instead of a proton and an electron as in the usual hydrogen).

i) Use Bohr's theory to calculate the energy levels of the muonic hydrogen. What is the energy of the ground state (in eV) ?

ii) Is the muon non-relativistic in the ground state of the muonic hydrogen ?

iii) What is the wavelength of a photon emitted in a transition between the first excited state and the ground state ? Which kind of photon is it (radio, microwave, visible, ultraviolet, X-ray, γ-ray, ...) ?

iv) Besides the mass, muons differ from electron by the fact that they "decay" into an electron and two massless, chargeless particles called "neutrinos". The lifetime for the decay is about $2 \times 10^{-6} s$. Does the muon have the time to orbit the proton several times before decaying when it is in the ground state of muonic hydrogen ?

B. Operator wizardry

i) Show that the operator $\hat{p} = -i\hbar d/dx$ is an hermitian operator.

ii) Compute

$$e^{-i\frac{y}{\hbar}\hat{p}} f(x). \tag{1}$$

Hint: expand the exponential and remember the Taylor series expression.

iii) Show that $\Psi(x,t) = e^{-i\hat{H}t/\hbar}\Psi(x,0)$ satisfies the Schrödinger equation. It is said that $\hat{p}$ generates space translations (from item ii) and $\hat{H}$ generates time translations (by item iii).