QUANTUM PHYSICS I

PROBLEM SET 4 - Solution

A. Blackbody radiation and the temperature of the Earth

i) We use the change of variables $x = h\nu/kT$ to arrive at

$$R_T = \frac{c}{4}\rho_T(\nu) = \int_0^\infty \frac{2\pi h\nu^2}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1} \tag{1}$$

$$= \frac{2\pi(kT)^4}{c^2h^3}\underbrace{\int_0^\infty \frac{x^3}{e^x-1}}_{T\ independent\ constant} \tag{2}$$

Using that

$$\int_0^\infty \frac{x^3}{e^x-1} = \frac{\pi^4}{15} \tag{3}$$

we find

$$R_T = \frac{2\pi^5k^4}{15c^2h^3}T^4 \tag{4}$$

and $\sigma = 2\pi^5/(15c^2h^3)$.

ii) The total power radiated by the sun is the product of the power radiated per unit area times the total surface area of the sun:

$$total\ power = 4\pi R_s^2\sigma T_s^4 \tag{5}$$

iii) The light emitted by the sun spreads in all directions and only a small fraction reaches the Earth. When the light reaches a distance r from the sun, it is spread over an area of $4\pi r^2$. The Earth occupies only an area of πR_E^2 (its cross section) of this surface. In addition, a fraction of a of the light shinning on the Earth surface is reflected and $1-a$ is absorbed. We find then

$$power\ absorbed = 4\pi R_s^2\sigma T_s^4\frac{\pi R_E^2}{4\pi r^2}(1-a). \tag{6}$$

iv) In equilibrium

$$4\pi R_s^2\sigma T_s^4\frac{\pi R_E^2}{4\pi r^2}(1-a) = 4\pi R_E^2\sigma T_E^4. \tag{7}$$

Solving for T_E we find

$$T_E = (1-a)^{1/4}\sqrt{\frac{R_s}{2r}}T_s \approx (1-0.3)^{1/4}\sqrt{\frac{7.10^8m}{300.10^9m}}5800K \approx 251\ K \approx -22\ C. \tag{8}$$

This calculation would seem more impressive if we input the Earth's average temperature $15C$ and compute the sun's temperature. The result comes out right to within a few percent. The difference between the temperature calculated here and the correct value ($15C$) is mostly due to the greenhouse effect.
