QUANTUM PHYSICS I

PROBLEM SET 1

due September 17, before class

A. Exercise your math muscles

1) compute i^i
2) compute $e^{i\pi/2}$
3) find the general solution to

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x), \tag{1}$$

for $E > 0$.

4) what is the solution to the problem above satisfying the conditions

$$\psi(0) = 1, \qquad \frac{d\psi(x)}{dx}|_{x=0} = 0 \text{ ?} \tag{2}$$

B. A first look at the Uncertainty Principle

Consider a particle described at some particular instant of time by the wave function $\psi(x) = Ae^{-ax^2}$.
1) Determine A so ψ is normalized.
2) Compute $\langle x\rangle$, $\langle x^2\rangle$ and $\sigma_x^2 = \langle(x - \langle x\rangle)^2\rangle$.
3) Compute $\langle p\rangle$, $\langle p^2\rangle$ and $\sigma_p^2 = \langle(p - \langle p\rangle)^2\rangle$.
4) Show that by changing a one can make either σ_x^2 or σ_p^2 small, but not both at the same time. Compute $\sigma_x\sigma_p$.

C. Ehrenfest's theorem

Prove that

$$\frac{\partial}{\partial t}\langle p\rangle = \int_{-\infty}^{\infty} dx\ \Psi(x,t)^* \left(-\frac{\partial V(x)}{\partial x}\right)\Psi(x,t). \tag{3}$$

This result is one way to show that, under certain circumstances, macroscopic objects obey Newton's law $F = ma$. Describe in words the connection of the formula above with Newton's law.