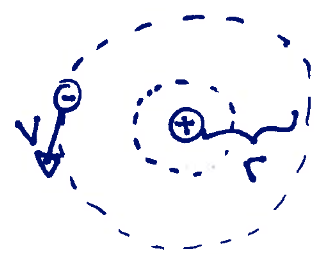
BOHR ATOM

The Bohr model of the atom is a combination of the Rutherford "planetary" model (nucleus at the center, electrons orbiting around it) with some of the early quantum mechanical ideas. It's based on a few points:

1. only orbits with angular momentum equal to a multiple of $\hbar$ are allowed

$$\text{(i)} \qquad L = mvr = n\hbar \;, \quad n = 1, 2, \ldots$$



hydrogen atom in the Bohr picture

2. classical mechanics is valid in each orbit (except the electron won't radiate)

$$\text{(ii)} \qquad F = \underbrace{\frac{e^2}{4\pi\epsilon_0 r^2}}_{\text{Coulomb force}} = m \underbrace{\frac{v^2}{r}}_{\text{centripetal acceleration}}$$

Eliminating v in (i) and plugging it in (ii)

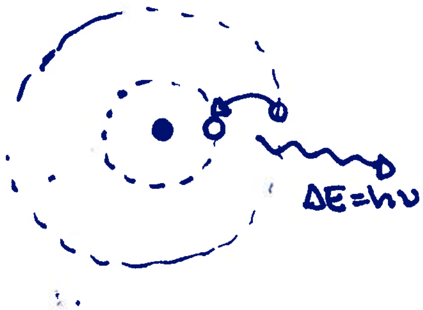
$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 \Rightarrow r = \frac{4\pi\epsilon_0\hbar^2}{me^2} n^2$$

The energy of each state is

$$E_n = \frac{mv^2}{2} - \frac{e^2}{4\pi\epsilon_0 r_n} = \frac{e^2}{8\pi\epsilon_0 r_n} - \frac{e^2}{4\pi\epsilon_0 r_n} = -\frac{e^2}{8\pi\epsilon_0 r_n}$$

$$= -\frac{e^2}{8\pi\epsilon_0}\frac{me^2}{4\pi\epsilon_0\hbar^2}\frac{1}{n^2} = -\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{1}{n^2} \simeq -\frac{13.6\,\text{eV}}{n^2}$$

3. The electron emits or absorbs a photon with frequency $\nu = \Delta E/h$ where ΔE is the difference of the energy of the initial and final states



$$\Delta E = \frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\Downarrow$$

$$\lambda = \frac{c}{\nu} = \frac{hc}{\Delta E} = \frac{2\hbar^3 c}{2\pi m}\left(\frac{4\pi\epsilon_0}{e^2}\right)^2 \frac{n_i^2 n_f^2}{n_i^2 - n_f^2}$$

These wavelengths are observed in the spectrum of hydrogen.

The Bohr model, despite its successes, has some shortcomings:

- the angular momentum of the ground state is $L=0$, not $L=1$, as we will see using quantum mechanics.
- cannot be easily generalized to more complex atoms.
- cannot explain the intensities of the spectral lines.
- it is not a consistent, systematic theory of the world, just a mix-match of ad-hoc rules.

DE BROGLIE HYPOTHESIS

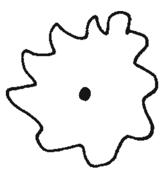
If light can be sometimes a wave (interference, diffraction, ...) and sometimes a particle (blackbody radiation, photoelectric effect, Compton scattering, ...), maybe material particles can also behave like a wave.

$$E = h\nu$$

$$p = \frac{h}{\lambda}$$

particle properties (E, p) — wave properties (ν, λ)

- "Explanation" for Bohr quantization rule:



integer number of wavelenght have to fit on an orbit: $\underbrace{n\lambda}_{n\frac{h}{p}} = 2\pi r \Rightarrow L = pr = n\hbar$

- wave properties of the electron verified by showing electron diffraction (Thomson (son), Davisson & Germer)

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<u>light</u>	<u>electron (and billiard balls, ...)</u>
Maxwell's eqs.	Quantum (wave) mechanics
$\Downarrow$ $\underbrace{\lambda}_{\text{wavelength}} \ll \underbrace{R}_{\text{size of trajectories}}$	$\Downarrow$ $\lambda \ll R$
geometrical optics, light rays	classical mechanics, particle trajectories