QUANTUM PHYSICS I - FALL 2007
MIDTERM EXAM II

I. MOMENTUM DISTRIBUTION

A particle of mass M is described at time $t = 0$ by the wave function:

$$\psi(x) = \begin{cases} A, & -L < x < L, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

a) Normalize $\psi(x)$. (*5 points*)
b) Calculate the momentum probability distribution. (*10 points*)
c) Calculate the position probability distribution. (*10 points*)
d) Consider the momentum distribution in the $L \to 0$ limit. How is it related to the uncertainty principle? (*10 points*)

Hint: The eigenfunctions of momentum $\psi_p(x) = \frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}}$ are normalized as

$$\int_{-\infty}^{\infty} dx \psi_p^*(x)\psi_{p'}(x) = \delta(p - p'). \tag{2}$$

II. BOUNCING FROM A HOLE

A particle of mass M moves in one dimension under the influence of the potential

$$V(x) = \begin{cases} V_0, & x < 0, \\ 0, & x > 0, \end{cases} \tag{3}$$

with $V_0 > 0$. If the particle is moving from the *left* towards the potential (form negative x to positive x)
a) Write down the Schroedinger equation for $x > 0$ and $x < 0$ and its solutions. (*10 points*)
b) What are the appropriate boundary conditions at $x = 0$? (*10 points*) c) What is the probability that the particle will continue moving towards $x = \infty$? (*20 points*)