QUANTUM PHYSICS I - FALL 2007

MIDTERM EXAM I

1 point for each item

A. Semi-classical quantization of the two-dimensional harmonic oscillator

Consider a particle of mass m revolving around a circular orbit under the influence of a potential of the form $V(\mathbf{r}) = m\omega^2\mathbf{r}^2/2$.

(a) Using the method that Bohr used to quantize the circular orbits in a hydrogen atom, *find the quantized energy levels of the particle.*
(b) Suppose the particle is charged and jumps from the first excited state to the ground state. *What is the frequency of the photon emitted during these transition ?*

B. Time evolution and probabilistic interpretation in the infinite square well

The wavefunction of an electron at time t = 0 in a one-dimensional infinite square well of width a is given by $\psi(x,0) = \sqrt{\frac{5}{7}}\psi_1(x) + \sqrt{\frac{2}{7}}\psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are the ground state and first excited stationary states of the system. ($\psi_n(x) = \sqrt{2/a}\ \sin(n\pi x/a), E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$, where $n = 1, 2, 3, \cdots$).

(a) Write down the wavefunction $\Psi(x,t)$ at time t in terms of $\psi_1(x)$ and $\psi_2(x)$.
(b) Suppose you measure the energy of the electron at time $t = 0$. Write down the possible values of the energy and the probability of measuring each.
(c) Calculate the expectation value of the energy in the state $\Psi(x,t)$ above.
(d) Repeat items (a) and (c) for the initial wavefunction $\psi(x,0) = \sqrt{\frac{5}{7}}\psi_1(x) - i\sqrt{\frac{2}{7}}\psi_2(x)$.

C. Expectation values, ...

A free particle at instant t is described by the wavefunction $\psi(x) = Ae^{-\alpha x^2}$.

a) Calculate the constant A so $\psi(x)$ is properly normalized.
b) Compute $\langle x\rangle, \langle p\rangle$ and $\langle E\rangle$ (E is the energy of the particle).

D. Photoelectric effect

Draw a (qualitative) graph of the current in the photoelectric effect

(a) as a function of the frequency of the incident light.
(b) as a function of the intensity of the light.

TURN PAGE

I. THINGS YOU MAY FIND USEFUL

$$
\begin{aligned}
\int_0^\infty dx e^{-\lambda x} &= \frac{1}{\lambda} \\
\int_0^L dx \sin(n\pi x/L)\sin(m\pi x/L) &= \frac{L}{2}\delta_{nm} \\
\int_0^L dx \cos(n\pi x/L)\cos(m\pi x/L) &= \frac{L}{2}\delta_{nm} \\
\int_{-\infty}^\infty dx e^{-\lambda x^2} &= \frac{\sqrt{\pi}}{\sqrt{\lambda}} \\
\int_{-\infty}^\infty dx x^2 e^{-\lambda x^2} &= \frac{\sqrt{\pi}}{2\sqrt{\lambda^3}} \\
\int_{-\infty}^\infty dx x^4 e^{-\lambda x^2} &= \frac{3\sqrt{\pi}}{4\sqrt{\lambda^5}}
\end{aligned}
\tag{1}
$$

$$
i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t).
$$