QUANTUM PHYSICS I
PROBLEM SET 5
due November 16th, before class

EXPECTATION VALUES ON HARMONIC OSCILLATOR STATIONARY STATES

Calculate the expectation values of x, p, x^2 and p^2 on the nth stationary state of the harmonic oscillator. Verify that the uncertainty principle is obeyed in each of them. Which state minimizes $\Delta x^2 \Delta p^2$?

COHERENT STATES

A coherent state of a harmonic oscillator is defined by

$$a_- |\lambda\rangle = \lambda |\lambda\rangle, \tag{1}$$

where λ is a complex number.

1. Prove that $[A, B^2] = [A, B]B + B[A, B]$ for any operators A and B.

2. Prove that $[a_-, a_+^n] = n a_+^{n-1}$

3. By using the expansion $e^{\lambda a_+} = 1 + \lambda a_+ + \frac{1}{2}\lambda^2 a_+^2 + \cdots$, show that

$$|\lambda\rangle = A e^{\lambda a_+} |0\rangle. \tag{2}$$

4. Show that $A = e^{-|\lambda|^2/2}$ for a properly normalized coherent state.

5. Write $|\lambda\rangle$ as

$$|\lambda\rangle = \sum_{n=0}^{\infty} c_n |n\rangle. \tag{3}$$

Find c_n.

Hint: if you are having trouble with bras and ket, just substitute $|n\rangle \to \psi_n$ everywhere.