QUANTUM PHYSICS I

PROBLEM SET 4

due October 24rd, before class

Static wave-packet

At some instant a particle has the wave function

$$\psi(x) = Ae^{-\alpha x^2}. \tag{1}$$

1. Normalize $\psi(x)$ and find $\langle x\rangle$ and $\langle p\rangle$(you've already done it last week).
2. Plot the probability density for the particle to be at x as a function of x.
3. Calculate and plot the probability density for the particle to have momentum p as a function of p.

Moving wave-packet

At some instant a particle has the wave function

$$\psi(x) = Ae^{-\alpha x^2 - ikx}. \tag{2}$$

1. Normalize $\psi(x)$ and find $\langle x\rangle$ and $\langle p\rangle$.
2. Plot the probability density for the particle to be at x as a function of x.
3. Calculate and plot the probability density for the particle to have momentum p as a function of p.

Schrödinger equation in momentum space

The Schrödinger equation for a free particle is

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2}\Psi(x,t). \tag{3}$$

1. Find the equation that the Fourier transform $c(p,t)$ of $\Psi(x,t)$ defined by

   $$c(p,t) = \int_{-\infty}^{\infty} dx \frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}}\Psi(x,t) \tag{4}$$

   satisfies. (Hint: apply $\int_{-\infty}^{\infty} e^{ipx/\hbar\cdots}$ to both sides of the Schrödinger equation).
2. Solve this equation (assume that the intial condition $c(p,0)$ is given). Does the momentum distribution change with time ?
3. Generalize the result above for a particle under the influence of a potential $V(x)$ (you should write your answer in terms of the Fourier transform $\tilde{V}(p) = \int_{-\infty}^{\infty} dxe^{ikx}V(x)$ of the potential). You will find a complicated integral equation instead of the algebraic equation found in the free case.