QUANTUM PHYSICS I

PROBLEM SET 2

due October 3rd, before class

Semi-classical quantization for a polynomial potential

In class we discussed the quantization of circular orbits of an electron around a nucleus considering only the Coulomb electrostatic force between the electron and the nucleus. Repeat the argument in the case of a modified interaction between electron and nucleus: assume the potential between them is $V(r) = \frac{\alpha}{r^n}$.

1. Write them the quantization of angular momentum rule (which is independent of the potential) and the "$F = ma$"' equation.
2. Determine the radii and energies of the allowed orbits

A first look at the Uncertainty Principle

Consider a particle described at some particular instant of time by the wave function $\psi(x) = Ae^{-ax^2}$.

1. Determine A so ψ is normalized.
2. Compute $\langle x \rangle$, $\langle x^2 \rangle$ and $\sigma_x^2 = \langle (x - \langle x \rangle)^2 \rangle$.
3. Compute $\langle p \rangle$, $\langle p^2 \rangle$ and $\sigma_p^2 = \langle (p - \langle p \rangle)^2 \rangle$.
4. Show that by changing a one can make either σ_x^2 or σ_p^2 small, but not both at the same time. Compute $\sigma_x \sigma_p$.

Ehrenfest's theorem

Prove that

$$\frac{\partial}{\partial t}\langle p \rangle = \int_{-\infty}^{\infty} \Psi(x,t)^* \left(-\frac{\partial V(x)}{\partial x} \right) \Psi(x,t). \qquad (1)$$

This result is one way to show that, under certain circumstances, macroscopic objects obey Newton's law $F = ma$. Describe in words the connection of the formula above with Newton's law.