PHYS 401 - QUANTUM PHYSICS - FINAL SOLUTION

PROBLEM I

1)
$$\begin{pmatrix} \langle a|\hat{H}|a\rangle & \langle a|\hat{H}|b\rangle \\ \langle b|\hat{H}|a\rangle & \langle b|\hat{H}|b\rangle \end{pmatrix} = \begin{pmatrix} E_0 & T \\ T & E_0 \end{pmatrix}$$

2)
$$\hat{H}|A\rangle = \Big[E_0\big(|a\rangle\langle a| + |b\rangle\langle b|\big) + T\big(|a\rangle\langle b| + |b\rangle\langle a|\big)\Big]\left[\frac{|a\rangle + |b\rangle}{\sqrt{2}}\right]$$
$$= \Big[E_0|a\rangle + E_0|b\rangle + T|a\rangle + T|b\rangle\Big]\,1/\sqrt{2}$$
$$= (E_0 + T)\left(\frac{|a\rangle + |b\rangle}{\sqrt{2}}\right)$$
$$= (E_0 + T)\,|A\rangle$$

$$\hat{H}|B\rangle = \Big[E_0\big(|a\rangle\langle a| + |b\rangle\langle b|\big) + T\big(|a\rangle\langle b| + |b\rangle\langle a|\big)\Big]\left[\frac{|a\rangle - |b\rangle}{\sqrt{2}}\right]$$
$$= \Big[E_0|a\rangle - E_0|b\rangle + T|b\rangle - T|a\rangle\Big]\,1/\sqrt{2}$$
$$= (E_0 - T)\left(\frac{|a\rangle - |b\rangle}{\sqrt{2}}\right)$$
$$= (E_0 - T)\,|B\rangle$$

3) From the calculation in 2) the eigenvalues are $E_0 + T$ and $E_0 - T$. Alternatively we can find the eigenvalues of the matrix H:

$$0 = \det\begin{pmatrix} E_0 - E & T \\ T & E_0 - E \end{pmatrix} = (E_0 - E)^2 - T^2 \Rightarrow E = E_0 \pm T$$

4) The eigenstates $|A\rangle$ and $|B\rangle$ evolve in time as $e^{-i(E_0+T)t/\hbar}|A\rangle$ and $e^{-i(E_0-T)t/\hbar}|B\rangle$. So $|\psi(t=0)\rangle = |a\rangle = \frac{|A\rangle + |B\rangle}{\sqrt{2}}$ will evolve as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}e^{-i(E_0+T)t/\hbar}|A\rangle + \frac{1}{\sqrt{2}}e^{-i(E_0-T)t/\hbar}|B\rangle$$
$$= \frac{e^{-i(E_0+T)t/\hbar} + e^{-i(E_0-T)t/\hbar}}{\sqrt{2}}|a\rangle + \frac{e^{-i(E_0+T)t/\hbar} - e^{-i(E_0-T)t/\hbar}}{\sqrt{2}}|b\rangle$$