QUANTUM PHYSICS I

PROBLEM SET 5

due November 15

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1) Find the matrix element of the hamiltonian of the anaharmonic oscillator in the position basis

$$\langle x|\frac{\hat{p}^2}{2m}+\frac{m\omega^2}{2}\hat{x}^2+\lambda\hat{x}^4|y\rangle \tag{1}$$

2) Find the matrix element of the hamiltonian of the anaharmonic oscillator in the momentum basis

$$\langle p|\frac{\hat{p}^2}{2m}+\frac{m\omega^2}{2}\hat{x}^2+\lambda\hat{x}^4|q\rangle \tag{2}$$

3) Find the matrix element of the hamiltonian below in the energy basis of the **harmonic** oscillator

$$\langle n|\frac{\hat{p}^2}{2m}+\frac{m\omega^2}{2}\hat{x}^2+g\hat{x}|m\rangle, \tag{3}$$

where $\hat{H}^{ho}|n\rangle = E_n^{ho}|n\rangle$, with $\hat{H}^{ho} = \frac{\hat{p}^2}{2m}+\frac{m\omega^2}{2}\hat{x}^2$.

4) Suppose the Hilbert space of a system is spawned by two orthonornaml vectors $|a\rangle$ and $|a\rangle$ and the hamiltonian is $\hat{H} = E_0|a\rangle\langle a| + E_1|b\rangle\langle b| + T_1|b\rangle\langle a| + T_2|a\rangle\langle b|$.

a) Show that the hamiltonian will be hermitian iff $T_1 = T_2^*$.

b) Compute the 2×2 matrix $\langle n|\hat{H}|b\rangle$, with $n = a, b$. Find the possible allowed values for the energy and the corresponding eigenstates.

c) Can you think of a physical system that can be approximated by the hamiltonian above ?