

QUANTUM PHYSICS I  
PROBLEM SET 2  
due September 29

**A. (Griffiths 1.18) When is quantum mechanics necessary ?**

In general, quantum mechanics is relevant when the de Broglie wavelength of the particle in question ( $h/p$ ) is comparable to the characteristic size of the system ( $d$ ). In thermal equilibrium a temperature  $T$ , the average kinetic energy of a particle is

$$\frac{p^2}{2m} = \frac{3}{2}kT, \quad (1)$$

where  $k$  is the Boltzmann constant, so the typical wavelength is

$$\lambda = \frac{h}{\sqrt{3mkT}}. \quad (2)$$

The purpose of this problem is to anticipate which systems will have to be treated quantum mechanically, and which can safely be described classically.

a) **Solids:** The lattice spacing in a typical solid is around  $d \approx 0.3nm$  (not very different from the size of an atom). Find the temperature below which the *free* electrons in a solid are quantum mechanical. Below which temperature are the nuclei in a solid quantum mechanical (use sodium as a typical example) ?

b) **Gases:** For what temperatures are the atoms in an ideal gas at pressure  $P$  quantum mechanical ? Hint: use the ideal gas law  $PV = NkT$  to deduce the interatomic spacing (the answer is  $T < (1/k)(h^2/3m)^{3/5}P^{2/5}$ ). Obviously we want  $m$  to be as small as possible and  $P$  as large as possible for the gas to show quantum behavior. Put in numbers for helium at atmospheric pressure. Is hydrogen in outer space (interatomic distance  $\approx 1cm$  and temperature  $\approx 3K$ ) quantum mechanical ?

**B. (Griffiths, 2.4) Uncertainty on the square well states**

Calculate  $\langle x \rangle$ ,  $\langle x^2 \rangle$ ,  $\langle p \rangle$  and  $\langle p^2 \rangle$  for the  $n$ th stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closer to the uncertainty limit ?

**C. (Griffiths, 2.5 and 2.6, sort of ...) When is the wave function phase relevant ?**

A particle on an infinite square well has an initial wave function that is an equal superposition of the two first states:

$$\Psi(x, 0) = A(\psi_1(x) + \psi_2(x)). \quad (3)$$

i) Normalize  $\Psi(x, 0)$ .

ii) Find  $\Psi(x, t)$  and  $|\Psi(x, t)|^2$ . To simplify the result, let  $\omega = \pi^2\hbar/(2ma^2)$ .

iii) Compute  $\langle x \rangle$  and notice it is oscillatory. What is the amplitude and angular frequency of this oscillation ?

Although the overall phase of the wave function is of no physical significance (it cancels out whenever you calculate a measurable quantity), the relative phase of the coefficients in eq. (3) does matter. For example, suppose we take

$$\Psi(x, 0) = A(\psi_1(x) + e^{i\phi}\psi_2(x)) \quad (4)$$

instead of eq. (3). Find

iv)  $\Psi(x, t)$ ,

v)  $|\Psi(x, t)|^2$  and

vi)  $\langle x \rangle$

and compare with the  $\phi = 0$  case.

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