Using Matlab functions

1. Create a text file with extension ".m" in your path
2. In the first line, define

   function <output> = <function name>(<input>)

3. In subsequent lines, calculate output.

Example: a text file called 'genH.m':

```
function H=genH(N)
H=diag(-2*ones(N,1))+diag(ones(N-1,1),1)+diag(ones(N-1,1),-1);
```

When called at the Matlab command prompt, we can now generate arbitrarily-large matrices w/ desired structure

```
>> genH(4)

ans =

    -2     1     0     0
     1    -2     1     0
     0     1    -2     1
     0     0     1    -2

>> genH(5)

ans =

    -2     1     0     0     0
     1    -2     1     0     0
     0     1    -2     1     0
     0     0     1    -2     1
     0     0     0     1    -2
```

"Diagonalization"

$\vec{X}_i$'s are orthonormal eigenvectors of Hermitian matrix $\vec{H}$ w/ eigenvalues λ_i.

What is $\vec{X}_i^{\dagger} \overleftrightarrow{H} \vec{X}_j$ $(i,j = 1, 2, \ldots)$?

Since $\overleftrightarrow{H}\vec{X}_j = \lambda_j \vec{X}_j$, $\vec{X}_i^{\dagger} \overleftrightarrow{H} \vec{X}_j = \vec{X}_i^{\dagger} \lambda_j \vec{X}_j = \lambda_j \vec{X}_i^{\dagger} \vec{X}_j = \lambda_j \delta_{ij} = \begin{bmatrix} \lambda_1 & 0 & \cdots \\ 0 & \lambda_2 & 0 \cdots \\ & & \ddots \end{bmatrix}$

So eigenvectors "diagonalize" the matrix!

Example

$\overleftrightarrow{H} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ eigenvalue: $\lambda_1 = 1$ $\lambda_2 = -1$

eigvector: $\vec{X}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\vec{X}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

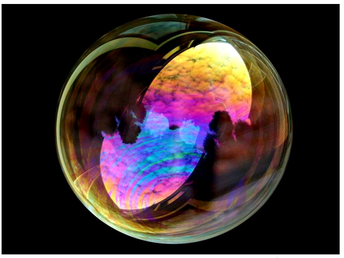
$i=1, j=1$: $X_1^{\dagger} H X_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1$

$i=1, j=2$: $X_1^{\dagger} H X_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = 0$

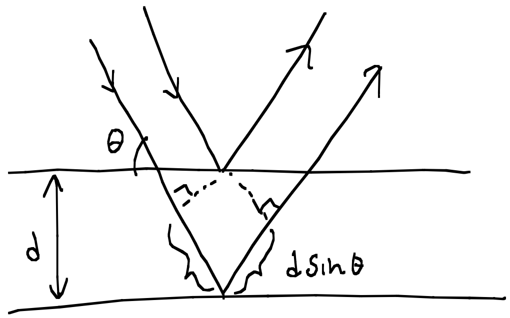
etc...

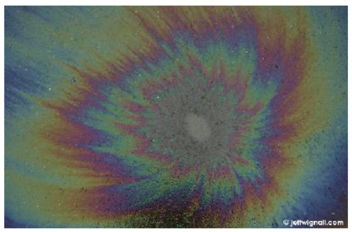
Matter Waves: Interference

EM waves: "Fabry-Perot"



soap bubble





oil slick

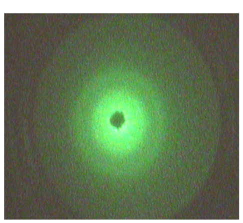
pathlength difference $= 2d\sin\theta = n\lambda$ $\quad n = 1, 2, 3, \ldots$ for constructive interference.

"Bragg's Law" $\rightarrow$ X-rays for $d \sim$ atomic spacing

electron waves: Davisson-Germer experiment (Nobel Prize, 1937)

DeBroglie: $p = \frac{h}{\lambda} \longrightarrow \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$ $\quad \left(E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}\right)$

graphite demo!



From Bragg's Law,

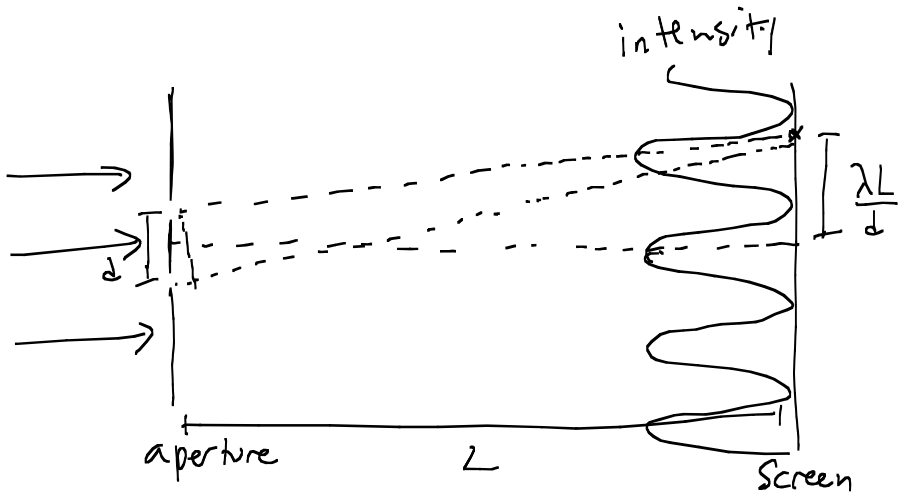
$$d = \frac{n\lambda}{2\sin\theta} = \frac{nh}{2\sin\theta\sqrt{2mE}} \sim \frac{h}{2\,{}^1\!/_{10}\sqrt{2m\,10^3\,\text{eV}}} = \frac{5.4\times10^{-15}\,\text{eV}\cdot\text{s}}{\sqrt{2\cdot 5\times10^{-16}\,\text{eV}\,\frac{s^2}{\text{cm}^2}\cdot 10^3\,\text{eV}}} \sim 2\,\text{Å}$$

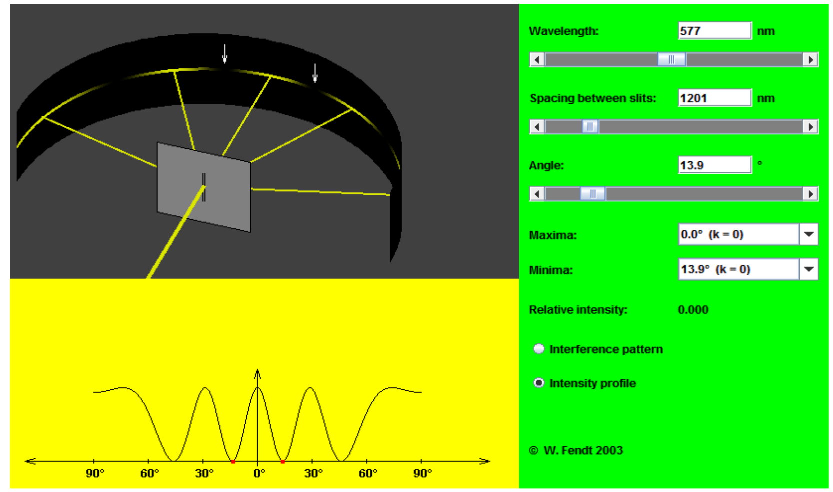
atomic scale spacing!

c.f. actual $\sim 3\,\text{Å}$!

$E = mc^2 \rightarrow m = E/c^2$ (mass of electron is 511 KeV$/c^2$)

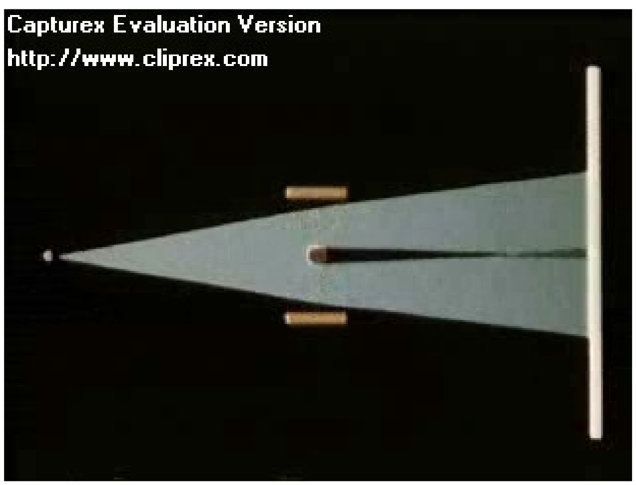
EM waves: Young's two-slit expt





http://www.walter-fendt.de/ph14e/doubleslit.htm

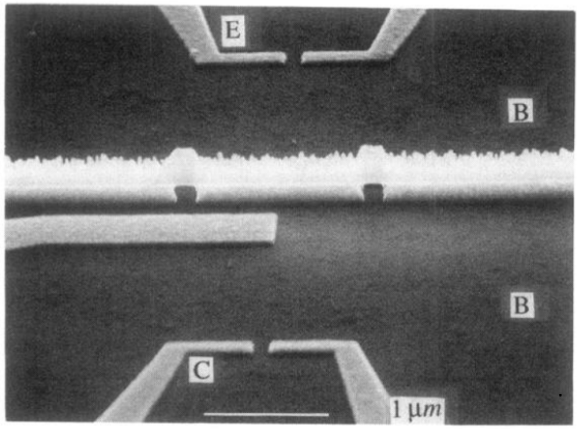
electrons: Biprism interference



electron fringes

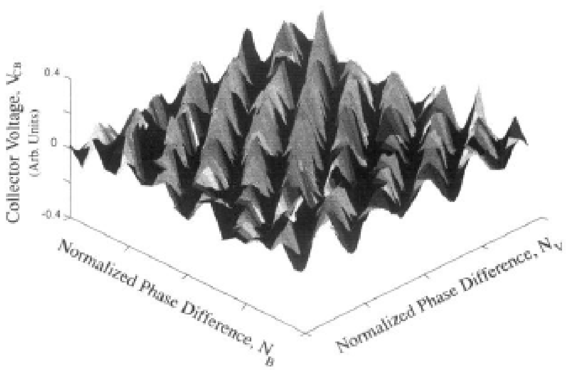
Electron Interferometry: Solid State (semiconductors)

two-slit:



A. Yacoby, M. Heiblum, V. Umansky, H. Shtrikman, and D. Mahalu

- Phys. Rev. Lett. 73, 3149 (1994)



An electronic Mach–Zehnder interferometer

Yang Ji, Yunchul Chung, D. Sprinzak, M. Heiblum, D. Mahalu & Hadas Shtrikman

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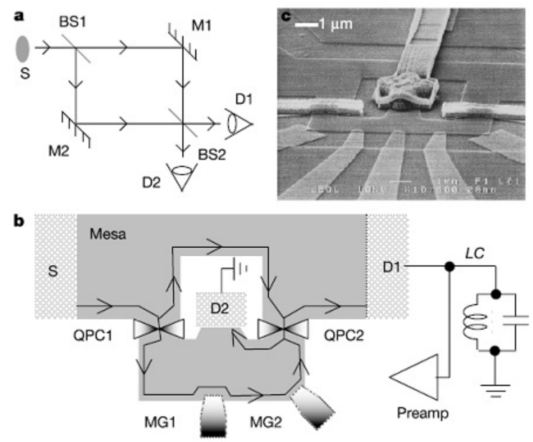

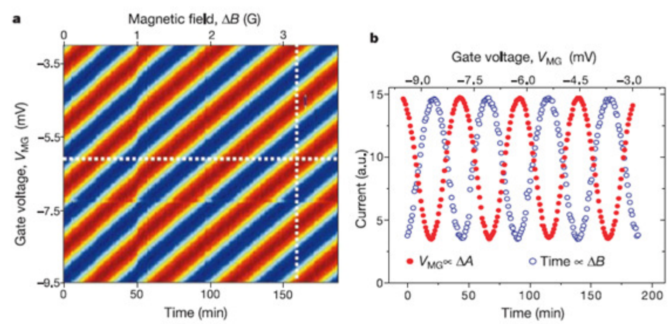

Potential Exam Topics

Ordinary diff. Eqs.
Partial diff. Eqs.
Separation of Variables
complex variables
taylor series
Fourier Transforms
orthonormality of fns
and vectors (incl. inner product)
"basis" functions and vectors

delta fns
gaussians
Uncertainty rel'n
Boundary conditions
finite differences
differential eigenvalue/eigenfunction problem
Matrix eigenvalue/ eigenvector problem
Hermitian matrices