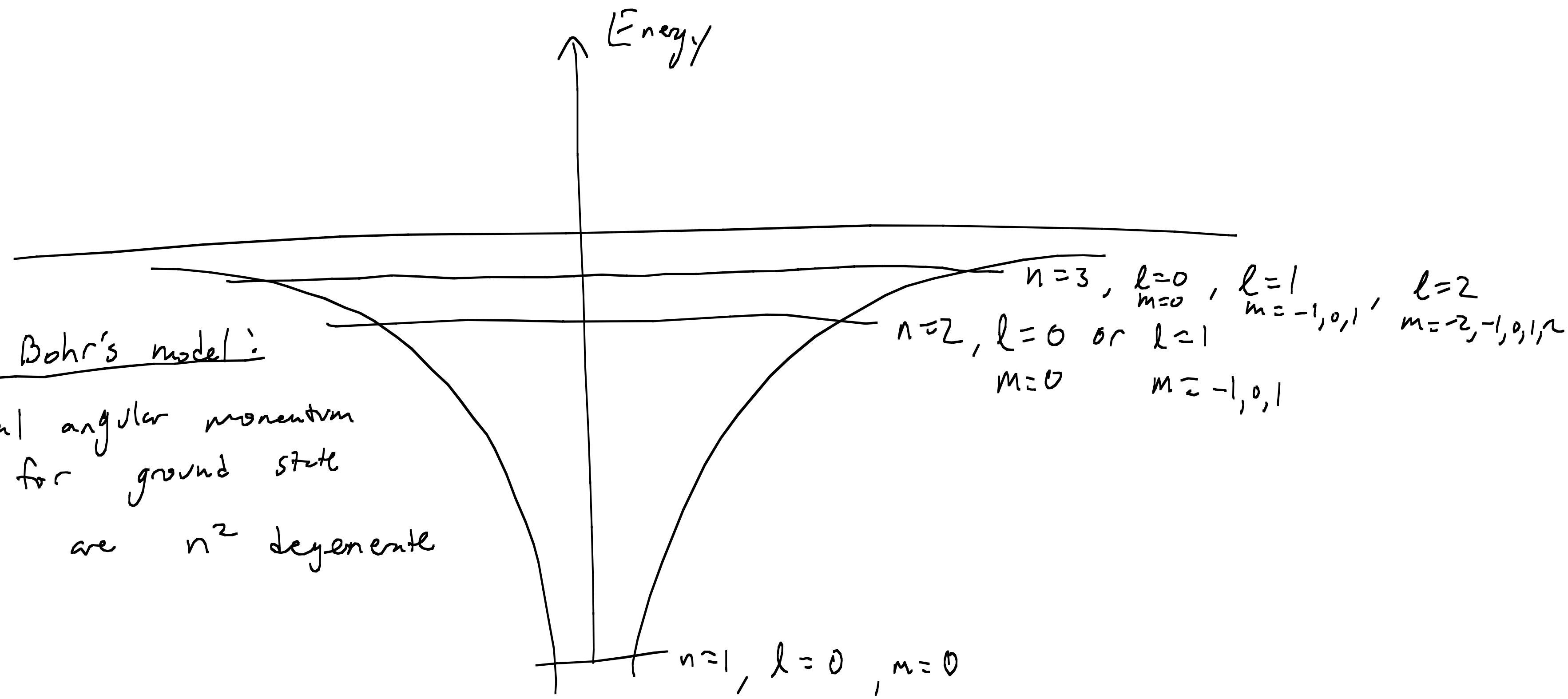
Energy

$$2n - \rho_0 = 0 \longrightarrow 2n - \left(\frac{2me^2}{4\pi\epsilon_0 \hbar^2 K}\right) = 0 \longrightarrow K = \frac{me^2}{4\pi\epsilon_0 \hbar^2 n}$$

so

$$E = -\frac{\hbar^2 K^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{me^2}{4\pi\epsilon_0 \hbar^2 n}\right)^2 = -\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{1}{n^2} \qquad n = 1, 2, 3 \ldots$$

(Bohr's formula!)



Unlike Bohr's model:

1. Orbital angular momentum $l=0$ for ground state
2. States are n^2 degenerate

Wave functions

$$\psi_{n\ell m}(r,\theta,\phi) = R_{n\ell}(r)\, Y_{\ell m}(\theta,\phi)$$

$$R_{n\ell}(r) = \frac{u_{n\ell}(r)}{r} = \frac{\rho^{\ell+1} e^{-\rho}\, V_{n\ell}(\rho)}{r} \quad , \qquad \rho = Kr = \frac{me^2 r}{4\pi\epsilon_0 \hbar^2 n} = \frac{r}{a_0 n}$$

$$a_0 \equiv \frac{4\pi\epsilon_0 \hbar^2}{me^2} \qquad \text{Bohr radius}$$

$V_{n\ell}(\rho)$ is determined by sum of polynomial terms given by recursion

Splitting degeneracy

electron with orbital angular momentum

$$|\vec{L}| = \sqrt{\ell(\ell+1)}\,\hbar$$

has a classical magnetic moment

$$\mu = I \cdot \text{Area} = -ef \cdot \pi r^2 = -\frac{ev}{2\pi r}\pi r^2 = -\frac{evr}{2} = -\frac{e(mvr)}{2m} = -\frac{e\hbar}{2m}\frac{|\vec{L}|}{\hbar}$$

$$= -\mu_B \frac{|\vec{L}|}{\hbar}$$

"Bohr magneton"

Energy in magnetic field

$$E = -\vec{\mu}\cdot\vec{B} = -\mu_z B_z \qquad \left(\vec{\mu} = \mu_x\hat{x} + \mu_y\hat{y} + \mu_z\hat{z}, \text{ and } \vec{B} = B_z\hat{z}\right)$$

So if states w/ same n have different $\langle \mu_z \rangle$, their energy eigenvalues will shift differently in a magnetic field and degeneracy will be broken!

Orbital Angular momentum in z-direction

we expect $\langle \mu_z \rangle = -\mu_B \frac{\langle L_z \rangle}{\hbar}$

Based on our Knowledge $\hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$, $\hat{p}_y = \frac{\hbar}{i}\frac{\partial}{\partial y}$, $\hat{p}_z = \frac{\hbar}{i}\frac{\partial}{\partial z}$

By analogy, $\hat{L}_z = \frac{\hbar}{i}\frac{\partial}{\partial \phi}$

$$\langle L_z \rangle = \langle \psi_{nlm} | \frac{\hbar}{i}\frac{\partial}{\partial \phi} | \psi_{nlm} \rangle$$

$$= \frac{\hbar}{i} \langle e^{im\phi} | \frac{\partial}{\partial \phi} | e^{im\phi} \rangle = \hbar m$$

"magnetic quantum number"

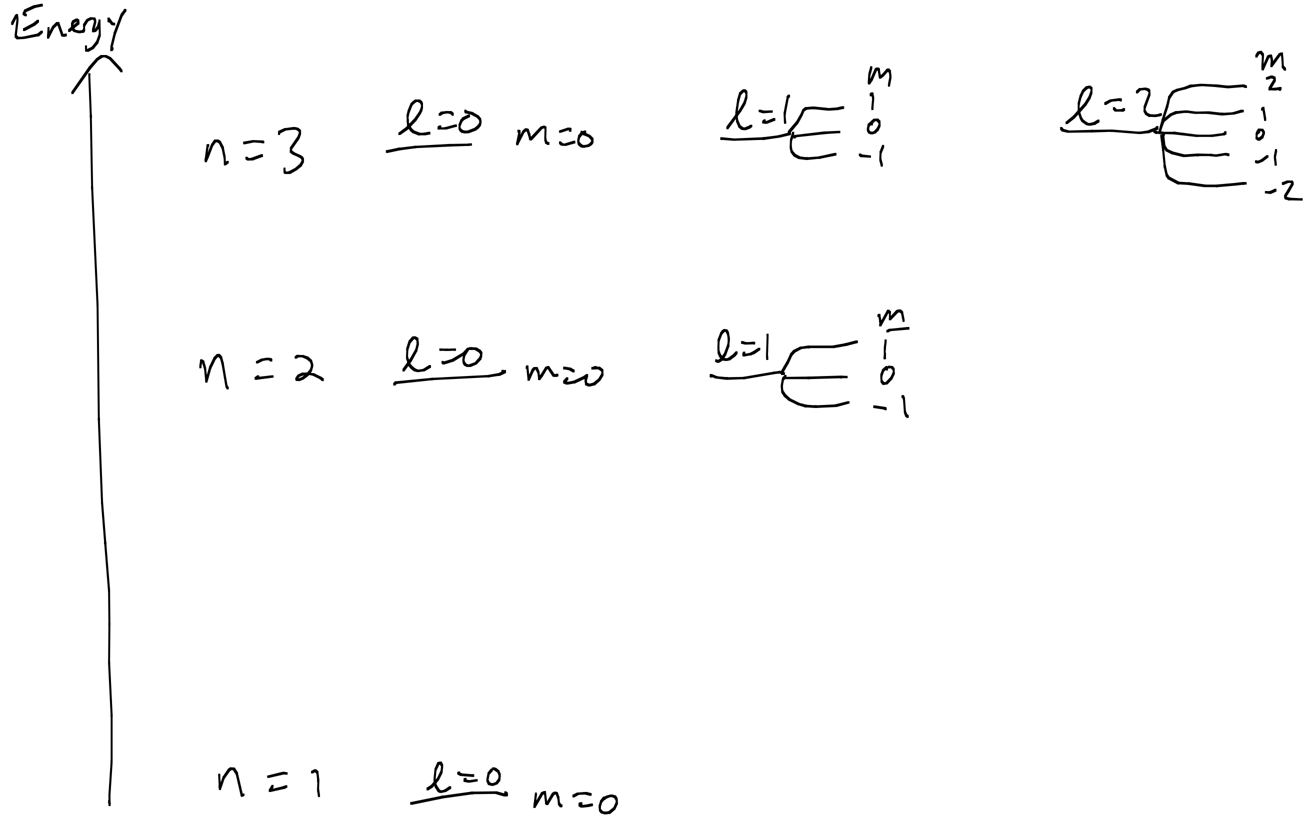
So "Zeeman energy" is

$$E = -\vec{\mu}\cdot\vec{B} = -\langle \mu_z \rangle B_z = \mu_B \frac{\langle L_z \rangle}{\hbar} B_z = \mu_B m B_z$$

Note: $\mu_B = 5.8 \times 10^{-5} \frac{eV}{T}$ (small in comparison to electronic transitions)

"Normal Zeeman Effect"

In a magnetic field, $\ell \neq 0$ levels split!



What effect does this have on optical spectra?

Not all transitions are allowed!