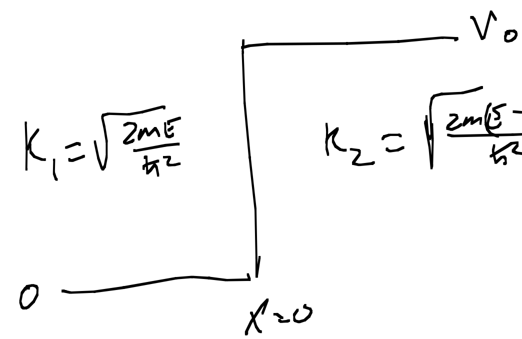
Example: Step Potential



$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

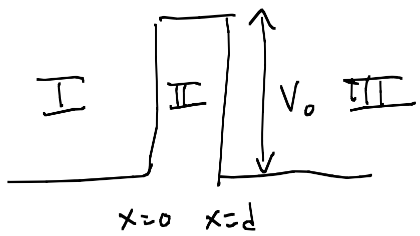
$$M = \begin{bmatrix} \left(\frac{1}{2} + \frac{k_2}{2k_1}\right) e^{i(k_2-k_1)x} & \left(\frac{1}{2} - \frac{k_2}{2k_1}\right) e^{-i(k_2+k_1)x} \\ \left(\frac{1}{2} - \frac{k_2}{2k_1}\right) e^{i(k_2+k_1)x} & \left(\frac{1}{2} + \frac{k_2}{2k_1}\right) e^{i(k_1-k_2)x} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{k_2}{2k_1} & \frac{1}{2} - \frac{k_2}{2k_1} \\ \frac{1}{2} - \frac{k_2}{2k_1} & \frac{1}{2} + \frac{k_2}{2k_1} \end{bmatrix}$$

So $M_{11} = \frac{1}{2} + \frac{k_2}{2k_1}$

$$T = \frac{k_R}{k_L} \frac{1}{|M_{11}|^2} = \frac{k_2}{k_1} \frac{1}{\left(\frac{1}{2} + \frac{k_2}{2k_1}\right)^2} = \frac{4k_2}{k_1\left(1 + \frac{k_2}{k_1}\right)^2} = \frac{4k_1k_2}{k_1^2\left(1 + \frac{k_2}{k_1}\right)^2} = \frac{4k_1k_2}{(k_1+k_2)^2}$$

c.f. previous result

"<u>Single barrier</u>"



what is $T(E)$?

$$\left(K_{I} = K_{III} = \sqrt{\frac{2mE}{\hbar^2}}, \quad K_{II} = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \right)$$

for each interface, construct

$$\begin{bmatrix} \left(\frac{1}{2} + \frac{k_2}{2k_1}\right) e^{i(k_2-k_1)x} & \left(\frac{1}{2} - \frac{k_2}{2k_1}\right) e^{-i(k_2+k_1)x} \\ \left(\frac{1}{2} - \frac{k_2}{2k_1}\right) e^{i(k_2+k_1)x} & \left(\frac{1}{2} + \frac{k_2}{2k_1}\right) e^{i(k_1-k_2)x} \end{bmatrix}$$

(where k_1 is to left of interface, k_2 is to right.)

$$\begin{bmatrix} \left(\frac{1}{2} + \frac{k_{II}}{2k_{I}}\right) & \left(\frac{1}{2} - \frac{k_{II}}{2k_{I}}\right) \\ \square & \square \end{bmatrix} \begin{bmatrix} \left(\frac{1}{2} + \frac{k_{III}}{2k_{II}}\right) e^{i(k_{III}-k_{II})d} & \square \\ \left(\frac{1}{2} - \frac{k_{III}}{2k_{II}}\right) e^{i(k_{III}+k_{II})d} & \square \end{bmatrix} = \begin{bmatrix} \underline{\underline{M_{11}}} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

@ x=0   only need these elements!   @ x=d

$$M_{11} = \left(\frac{1}{2} + \frac{k_{II}}{2k_{I}}\right)\left(\frac{1}{2} + \frac{k_{III}}{2k_{II}}\right) e^{i(k_{III} - k_{II})d} + \left(\frac{1}{2} - \frac{k_{II}}{2k_{I}}\right)\left(\frac{1}{2} - \frac{k_{III}}{2k_{II}}\right) e^{i(k_{III} + k_{II})d}$$

$$= e^{i(k_{III} - k_{II})d}\left(A + B e^{+2ik_{II}d}\right)$$

$$T = \left|\frac{1}{M_{11}}\right|^2 = \frac{1}{A^2 + B^2 + AB\left(e^{2ik_{II}d} + e^{-2ik_{II}d}\right)} = \frac{1}{A^2 + B^2 + 2AB\cos 2k_{II}d}$$

oscillating if k_{II} real $(E > V_0)$

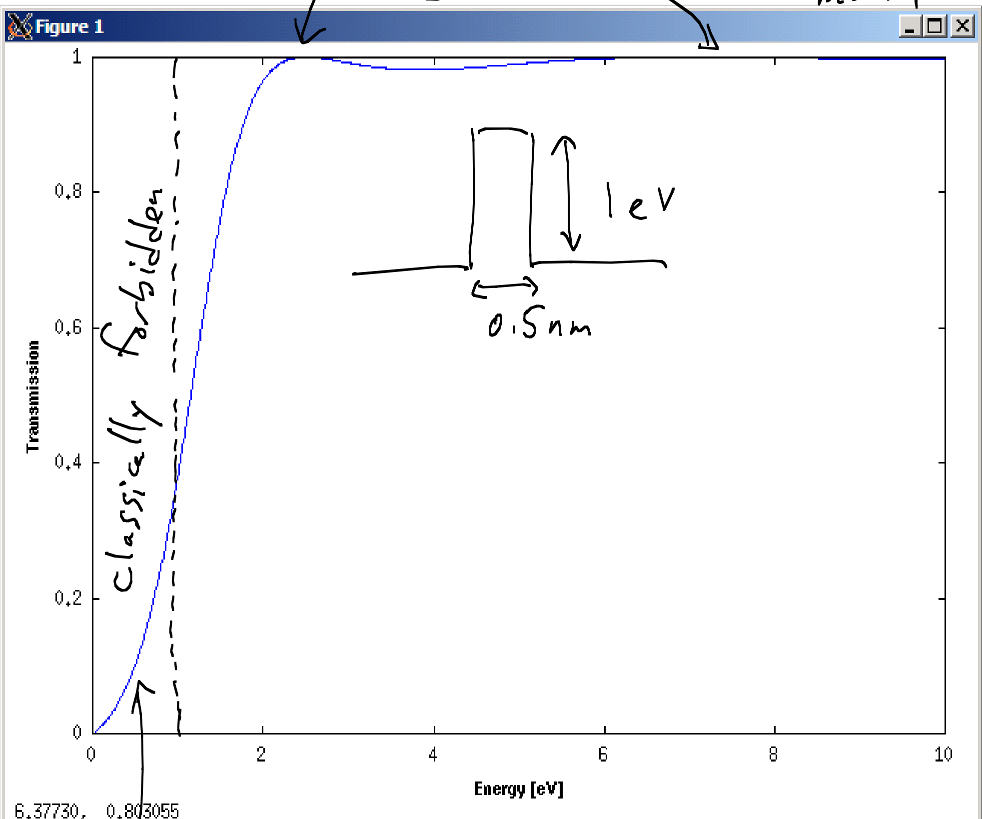
Note asymptotic behavior:

If $E \gg V_0$, $k_{I}, k_{III} \sim k_{II}$ so $A \to 1$ and $B \to 0$ so $T \to 1$

If E small, $k_{I}, k_{III} \to 0$ so A, B diverge and $T \to 0$

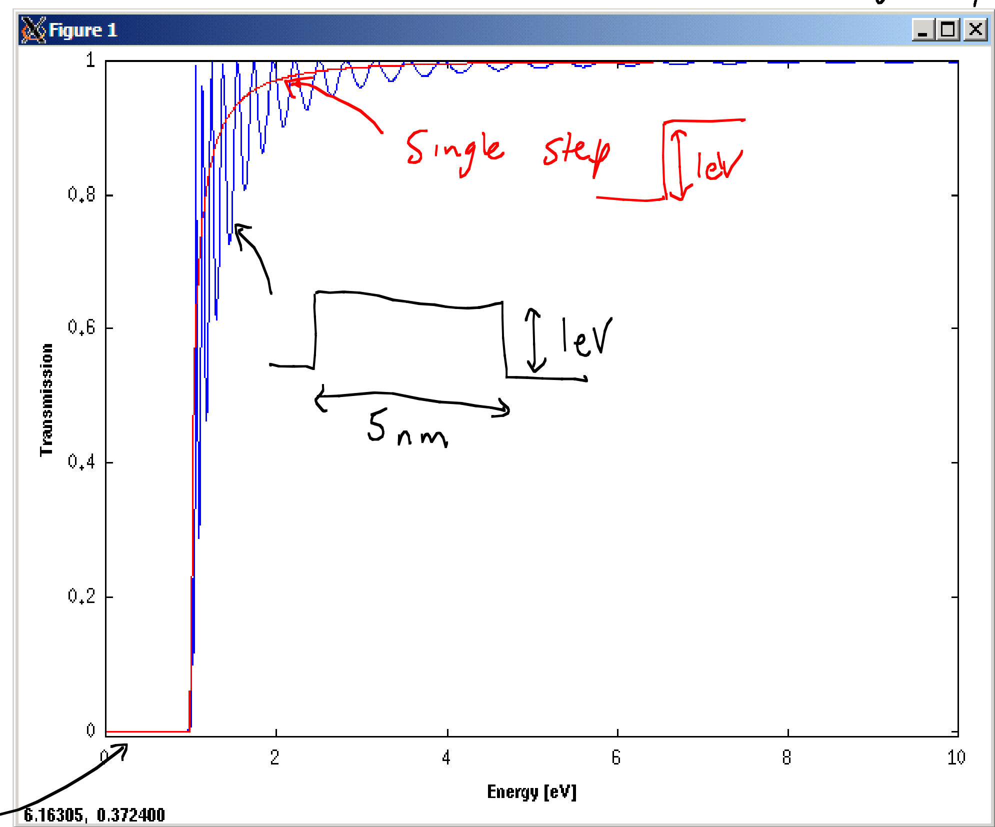
Results

perfect (T=1) transmission due to constructive interference upon multiple reflection from two interfaces (like Fabry-Perot!)



"tunneling" from evanescent wave in barrier

Making barrier wider decreases interference oscillation period and tends toward limit of single-step!



exponentially suppressed tunneling