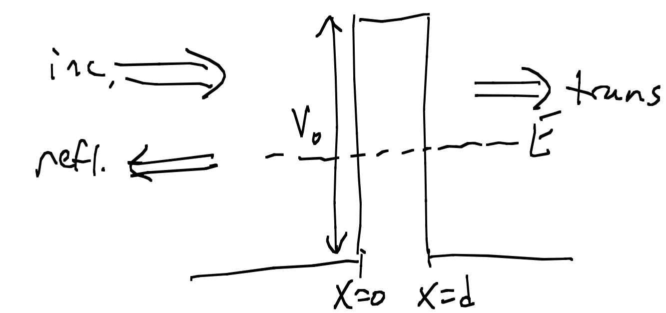
>1 interface



for $x<0$ and $x>d$:

$$-\frac{\hbar^2}{2m}\varphi'' = E\varphi \Rightarrow \varphi'' = -k^2\varphi, \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

for $0<x<d$:

$$-\frac{\hbar^2}{2m}\varphi'' + V_0\varphi = E\varphi \Longrightarrow \varphi'' = K^2\varphi, \quad K = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

I. find $\varphi(x)$ in all three regions

$$\varphi(x) = \begin{cases} \varphi_-(x) = e^{ikx} + re^{-ikx}, & x<0 \\ \varphi_b(x) = Ae^{Kx} + Be^{-Kx}, & 0<x<d \\ \varphi_t(x) = te^{ikx}, & x>d \end{cases}$$

II. Apply Boundary conditions

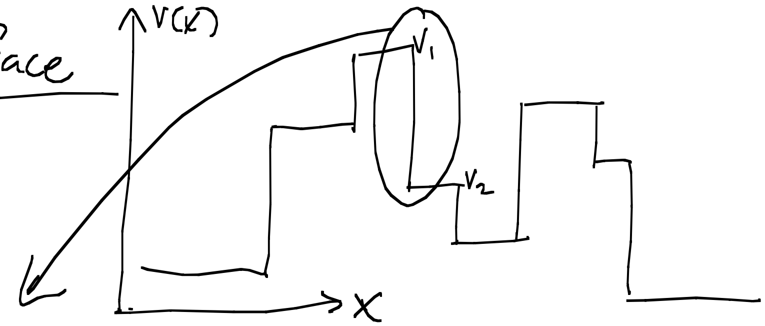
① $\varphi_-(0) = \varphi_b(0)$ ③ $\varphi_b(d) = \varphi_t(d)$

② $\varphi_-'(0) = \varphi_b'(0)$ ④ $\varphi_b'(d) = \varphi_t'(d)$

But this is tedious and not practical for complicated potentials w/ many interfaces!

III. Calculate $T = J_{trans}/J_{inc}$

Boundary Conditions at an arbitrary interface



$$Ae^{ik_1x} \Rightarrow \qquad Ce^{ik_2x} \Rightarrow$$

$$+Be^{-ik_1x} \Leftarrow \qquad +De^{-ik_2x} \Leftarrow$$

$$k_1 = \sqrt{\frac{2m(E-V_1)}{\hbar^2}} \qquad k_2 = \sqrt{\frac{2m(E-V_2)}{\hbar^2}}$$

bdry

ψ continuous:
$$\left. Ae^{ik_1x} + Be^{-ik_1x} \right|_{bdry} = \left. Ce^{ik_2x} + De^{-ik_2x} \right|_{bdry}$$

ψ' continuous:
$$\left. ik_1Ae^{ik_1x} - ik_1Be^{-ik_1x} \right|_{bdry} = \left. ik_2Ce^{ik_2x} - ik_2De^{-ik_2x} \right|_{bdry}$$

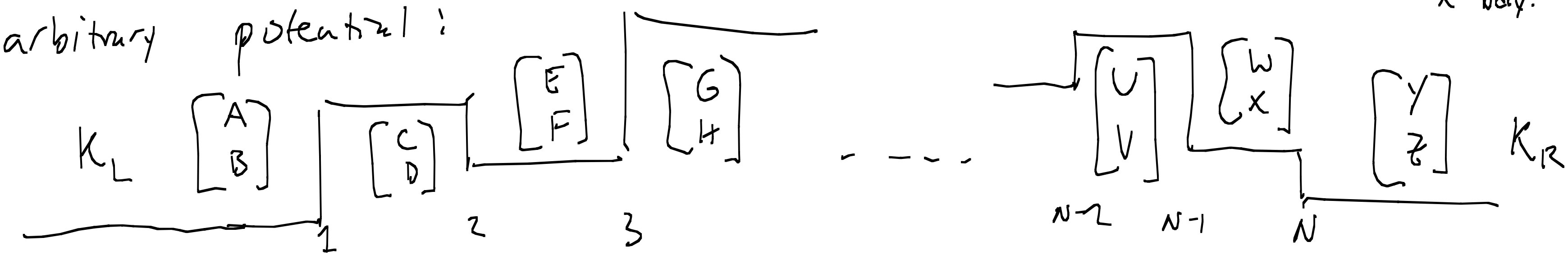
A matrix equation:

$$\left.\begin{bmatrix} e^{ik_1x} & e^{-ik_1x} \\ ik_1e^{ik_1x} & -ik_1e^{-ik_1x} \end{bmatrix}\right|_{bdry} \begin{bmatrix} A \\ B \end{bmatrix} = \left.\begin{bmatrix} e^{ik_2x} & e^{-ik_2x} \\ ik_2e^{ik_2x} & -ik_2e^{-ik_2x} \end{bmatrix}\right|_{bdry} \begin{bmatrix} C \\ D \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \frac{1}{2} e^{-ik_1 x} & -\frac{1}{2}\frac{i}{k_1} e^{-ik_1 x} \\ \frac{1}{2} e^{ik_1 x} & \frac{1}{2}\frac{i}{k_1} e^{ik_1 x} \end{bmatrix} \begin{bmatrix} e^{ik_2 x} & e^{-ik_2 x} \\ ik_2 e^{ik_2 x} & -ik_2 e^{-ik_2 x} \end{bmatrix}\Bigg|_{\text{bdry}} \begin{bmatrix} C \\ D \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2} + \frac{k_2}{2k_1}\right) e^{i(k_2 - k_1)x} & \left(\frac{1}{2} - \frac{k_2}{2k_1}\right) e^{-i(k_1 + k_2)x} \\ \left(\frac{1}{2} - \frac{k_2}{2k_1}\right) e^{i(k_1 + k_2)x} & \left(\frac{1}{2} + \frac{k_2}{2k_1}\right) e^{-i(k_2 - k_1)x} \end{bmatrix}\Bigg|_{x = \text{bdry.}} \begin{bmatrix} C \\ D \end{bmatrix}$$

For arbitrary potential:



$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \text{interface 1} \end{bmatrix} \cdots \begin{bmatrix} \text{interface } N\text{-}1 \end{bmatrix} \begin{bmatrix} \text{interface } N \end{bmatrix} \begin{bmatrix} Y \\ Z \end{bmatrix} = \overleftrightarrow{M} \begin{bmatrix} Y \\ Z \end{bmatrix}$$

Transmission Coefficient

$$\begin{bmatrix} A = 1 \\ \\ B = r \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} y = t \\ \\ z = 0 \end{bmatrix}$$

$$1 = M_{11} t \quad \Rightarrow \quad t = \frac{1}{M_{11}}$$

$$r = M_{21} t \quad \Rightarrow \quad r = \frac{M_{21}}{M_{11}}$$

$$T = \frac{J_{trans}}{J_{inc}} = \frac{\frac{\hbar K_R}{m} |t|^2}{\frac{\hbar K_L}{m}} = \frac{K_R}{K_L} \left| \frac{1}{M_{11}} \right|^2 \quad , \quad R = \left| \frac{M_{21}}{M_{11}} \right|^2$$

"Recipe":

For a given piecewise-constant scattering potential:

1. Pick E
2. Construct 2x2 matrix for each interface from E, V_1, V_2, and position of interface x
3. Multiply them together (in proper order) to get $\hat{M}$
4. Calculate $T(E) = \frac{k_R}{k_L} \frac{1}{|M_{11}|^2}$
5. Go to #1 (repeat for different E)