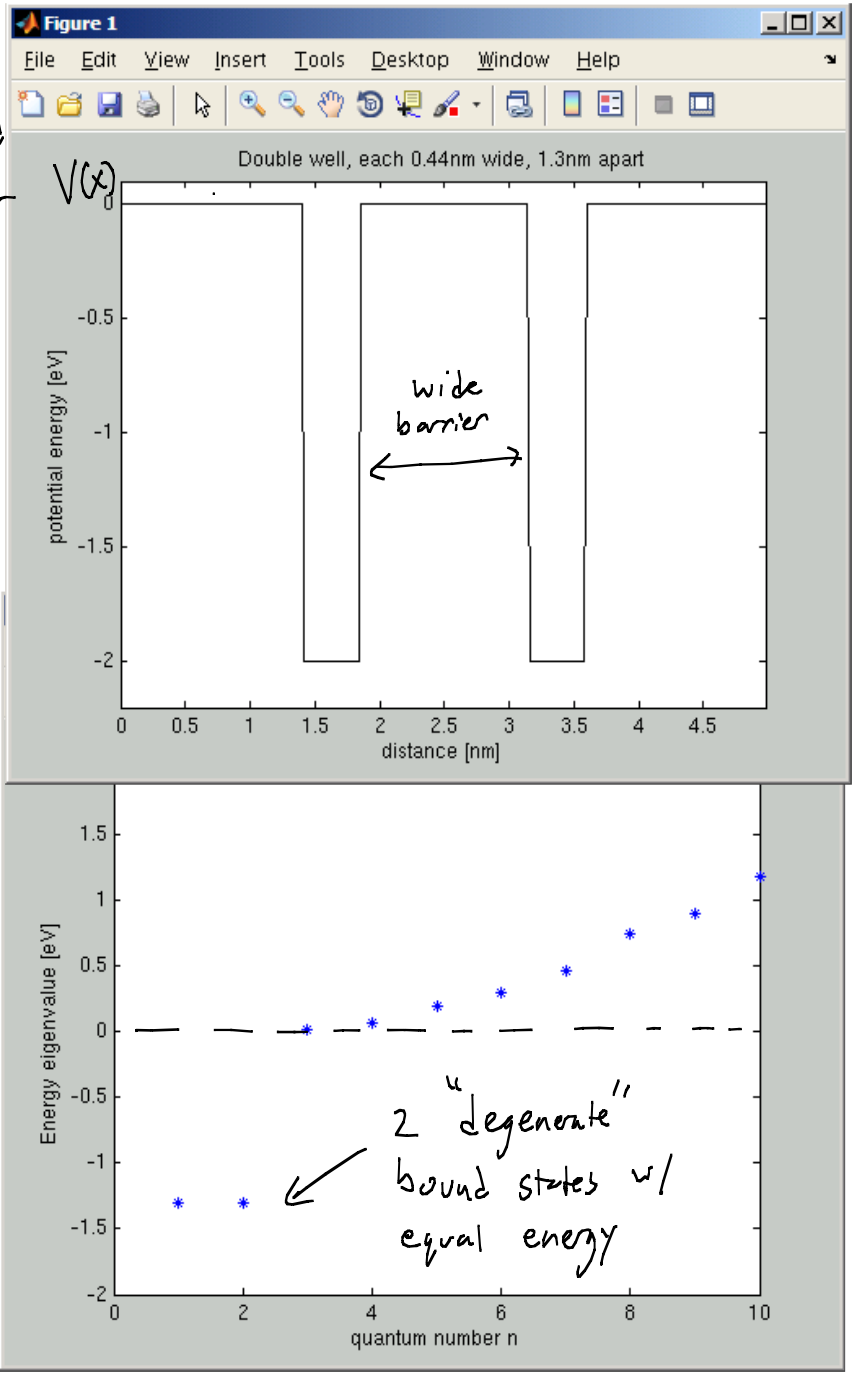
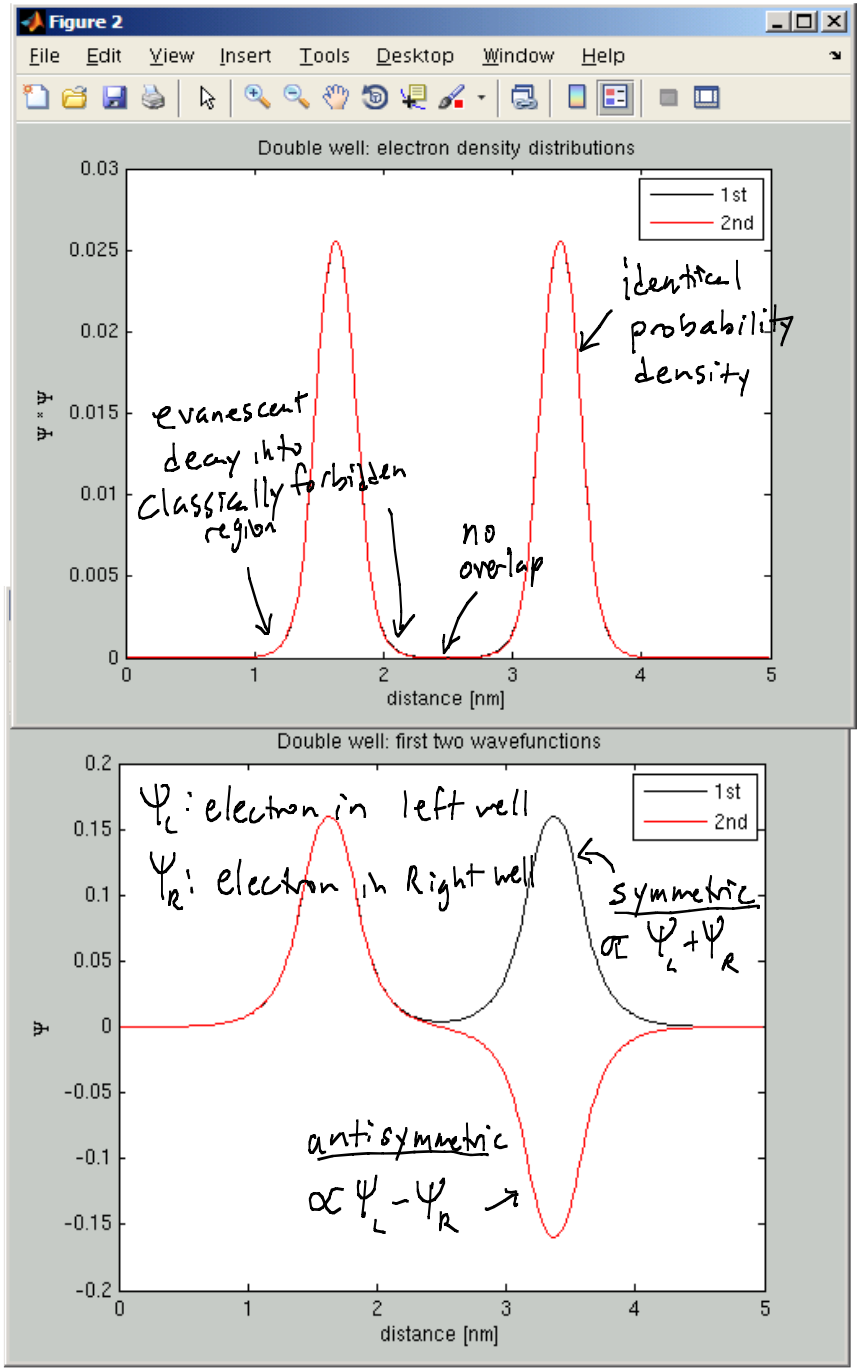
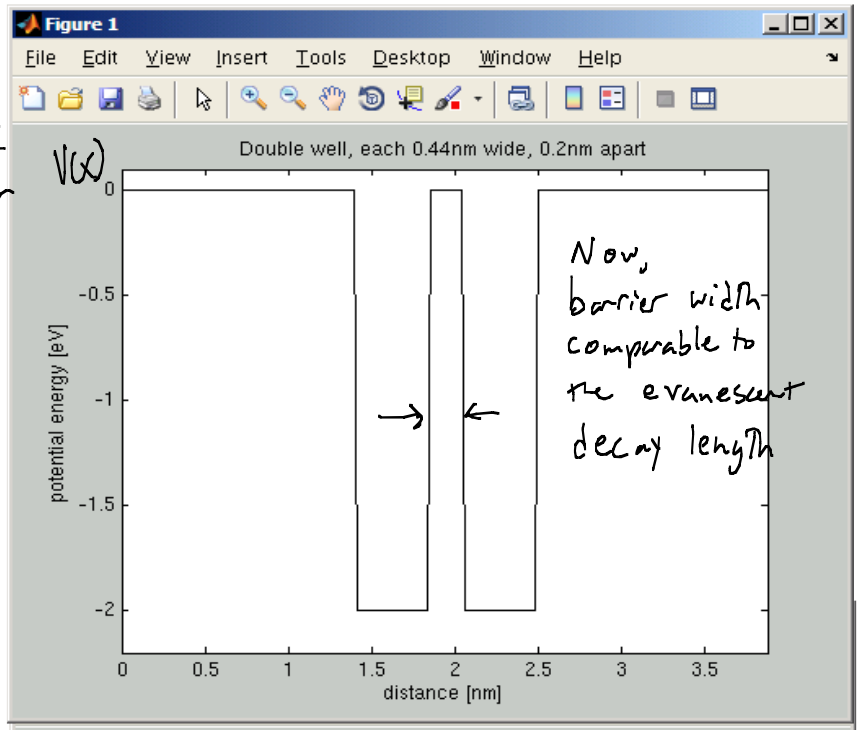
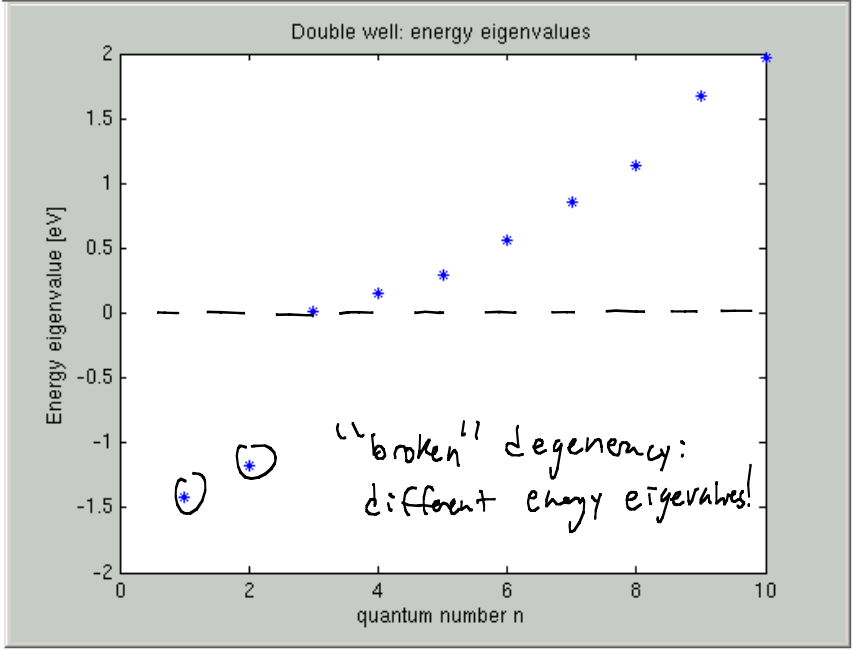
Two finite QWs, decoupled by wide barrier

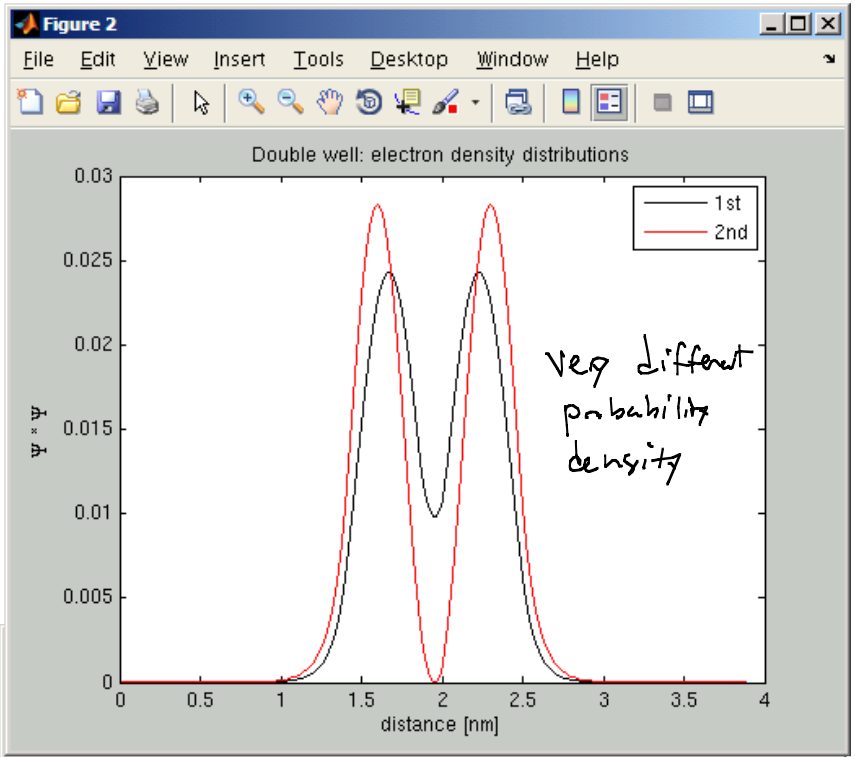


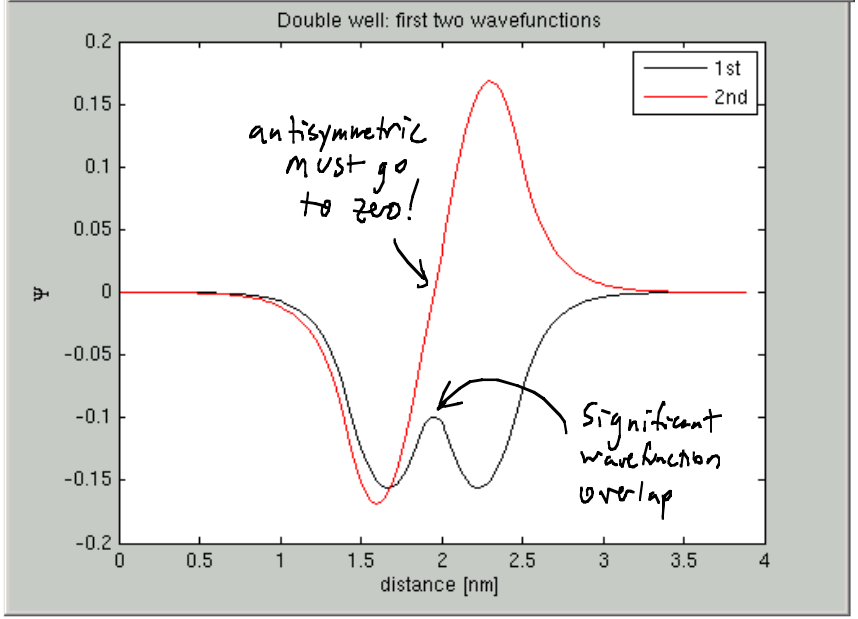


Two finite QWs, coupled by thin barrier

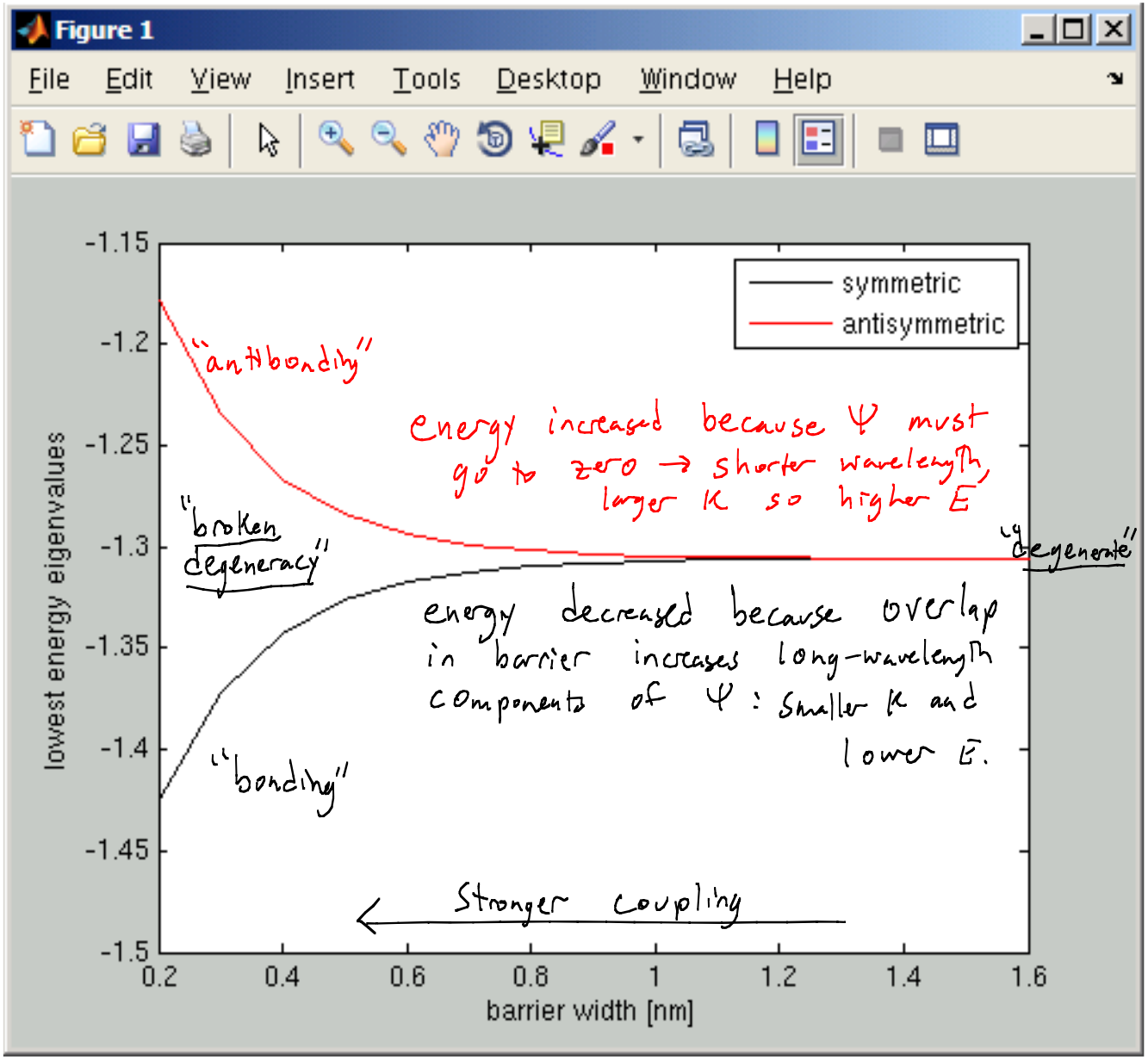




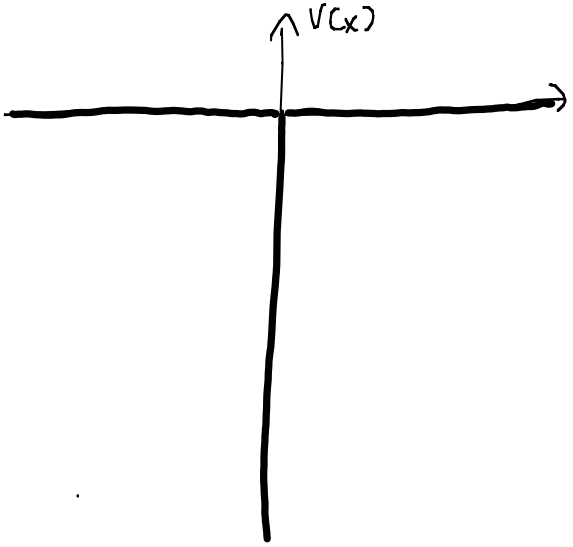




Two finite QWs: bound state "repulsion"



Exception to B.C. "φ' continuous"



Example: $V(x) = -\alpha\delta(x)$ $\left(\int \delta(x)\,dx = 1\right)$

$$-\frac{\hbar^2}{2m}\varphi'' - \alpha\delta(x)\varphi = E\varphi$$

$$-\frac{\hbar^2}{2m}\int_{-\epsilon}^{\epsilon}\varphi''\,dx - \alpha\int_{-\epsilon}^{\epsilon}\delta(x)\varphi\,dx = \int_{-\epsilon}^{\epsilon}E\varphi\,dx$$

$$\lim_{\epsilon\to 0}: \quad -\frac{\hbar^2}{2m}\left(\varphi'\Big|_{x=\epsilon} - \varphi'\Big|_{x=-\epsilon}\right) - \alpha\varphi(0) = 0$$

$$\varphi'\Big|_{x=\epsilon} - \varphi'\Big|_{x=-\epsilon} = -\frac{2m\alpha}{\hbar^2}\varphi(0) \qquad \underline{\text{NOT}} \text{ zero!}$$

Note: Since $\int_{-\infty}^{\infty}\delta(x)\,dx = 1$, since dx has units of length, $\delta(x)$ must have units of length^{-1}. Therefore, since $V(x)$ has units of energy, α has units of Energy·length.