

# Phys 401: Quantum Physics I

There is no magic in QM, and in any case this first course will not address any "quantum weirdness" emphasized in popularizations of the subject.

We need only accept that experiments show electrons behave as matter waves. The content of this course follows directly from this fact.

Even this unusual idea (that a physical object can be both a particle and a wave) is not unique to QM of electrons!

## Particle - Wave Duality: before advent of QM of electron

Electricity and Magnetism: Maxwell's equations + Lorentz force eqn.

light propagation: 1-d wave eqn for  $\vec{E}(z,t) = E_x(z,t)\hat{x}$

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \quad \rightarrow \quad \frac{\partial^2}{\partial z^2} E_x = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E_x \quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad \text{"speed of light"}$$

Solution:

$$E_x(z,t) = E_0 e^{i(kz - \omega t)} \quad \text{"plane wave"}$$

$$-k^2 E_x = -\frac{\omega^2}{c^2} E_x \quad \Rightarrow \quad \omega = kc \quad \text{"dispersion relation"}$$

$$\begin{array}{ccc} \text{"radial frequency"} & \text{"wavenumber"} & \\ \downarrow \omega = 2\pi f & \downarrow k = \frac{2\pi}{\lambda} & \Rightarrow \lambda f = c \\ \text{"frequency"} & \text{"wavelength"} & \end{array}$$

Interference, diffraction phenomena experimentally show that light is a wave as described above!

## Energy and momentum transport in E+M

Fields carry Momentum:  $\vec{P} = \mu_0 \epsilon_0 \vec{S} = \epsilon_0 (\vec{E} \times \vec{B})$

"Poynting's vector"

For plane wave,  $\vec{E} \perp \vec{B}$  and in phase,  $|\vec{B}| = \frac{|\vec{E}|}{c}$

$\swarrow$  proportional to Intensity

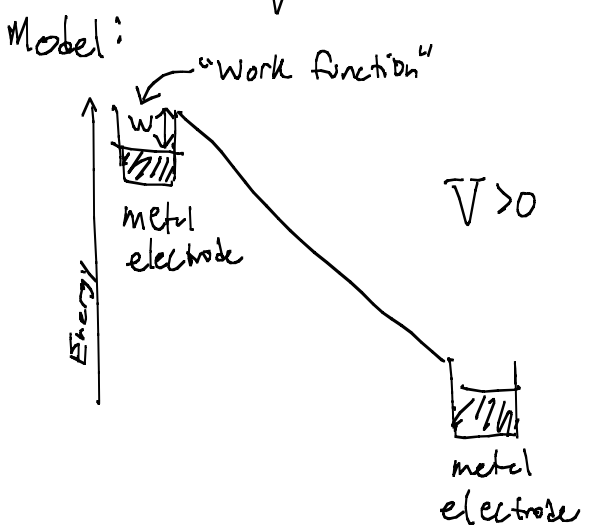
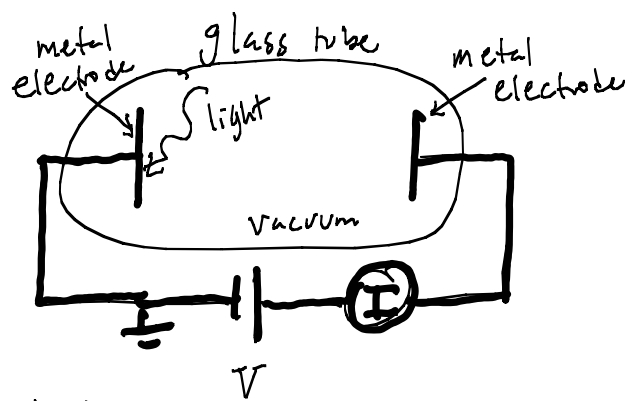
$$p = \epsilon_0 \frac{|\vec{E}|^2}{c} = \frac{U}{c}$$

Fields carry Energy:  $U = \frac{1}{2} (\epsilon_0 |\vec{E}|^2 + \frac{1}{\mu_0} |\vec{B}|^2) = \epsilon_0 |\vec{E}|^2 = pc$

However:

Experiments done prior to QM (< late 1920s) conclusively show that this prediction of classical E+M (Energy + momentum carried by light prop. to intensity) does not provide the relevant scale in light-matter interactions!

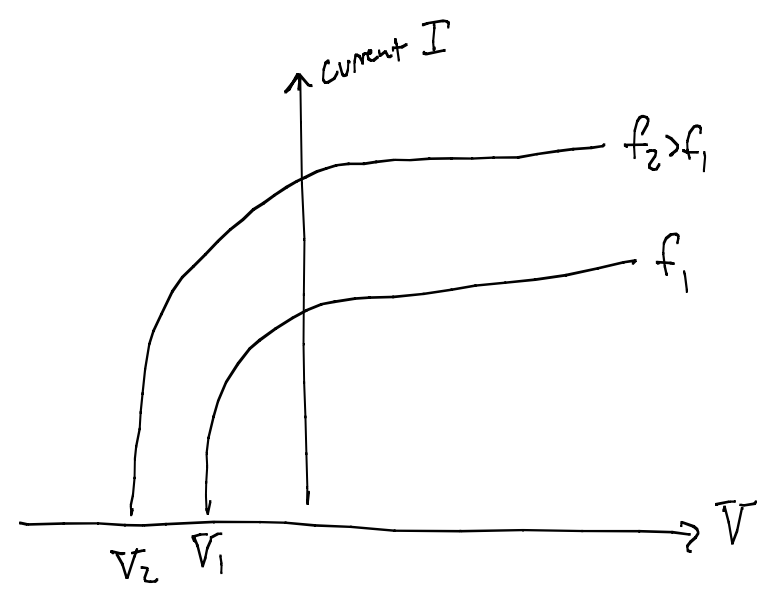
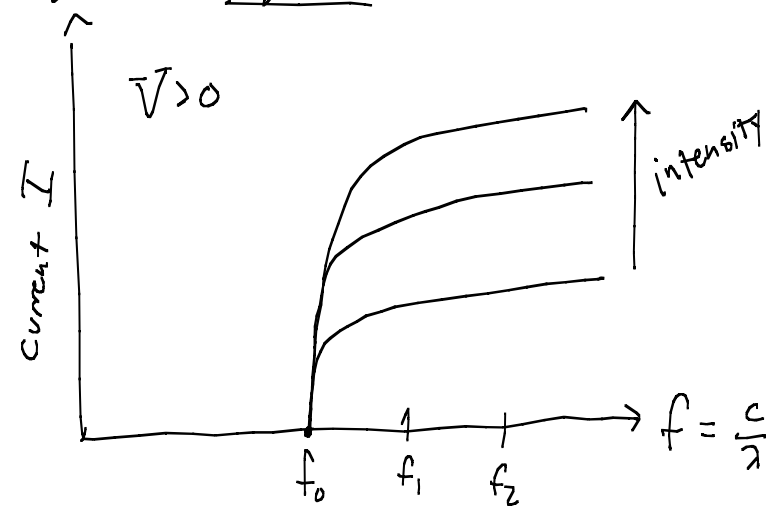
# Experiment #1: Photoelectric effect



Charge will flow if Energy absorbed from light  $E > W$  ("work function" of metal)  
 Classical Theory predicts we just need to increase the intensity!

(1887 H. Hertz)  
 (1902 Lenard)

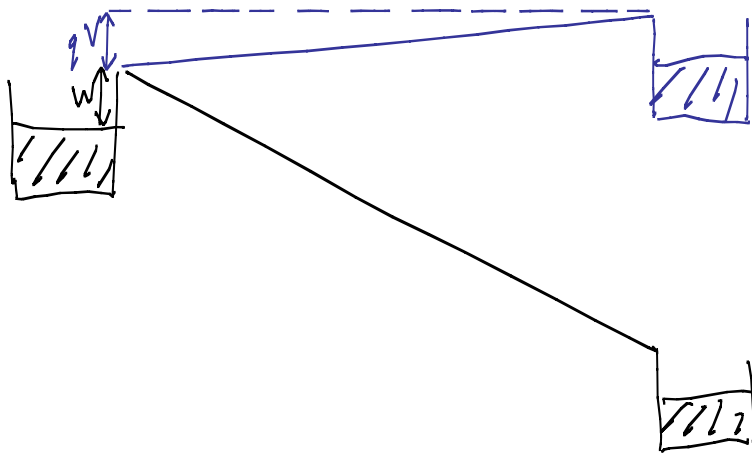
Experimental results show that classical prediction is wrong  $\rightarrow$  frequency more important than Intensity!



## Explanation (1905, Einstein)

EM waves come in discrete units of Energy proportional to frequency:

$$\underline{E = hf} \quad (\text{Note: } h \text{ has units of Energy} \cdot \text{time})$$



$V < 0$ : need  $E = hf > W + qV$  for current to flow

$V > 0$ : need  $E = hf > W$  for current to flow

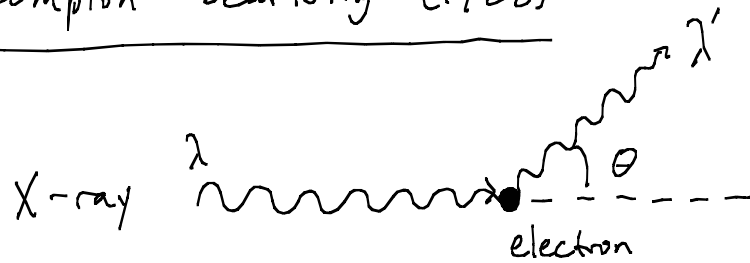
By repeating the experiment w/ different frequencies  $f_1$  and  $f_2$ , measuring thresholds  $V_1$  and  $V_2$ , we can solve for  $h$ :

$$\begin{aligned} hf_1 &= W + qV_1 \\ hf_2 &= W + qV_2 \end{aligned} \Rightarrow h = q \frac{V_1 - V_2}{f_1 - f_2}$$

Numerical value same as found for "Blackbody radiation" spectrum (Planck)!

(Note:  $E = hf$  is equivalent to  $E = \hbar\omega$  where  $\hbar = h/2\pi$  and  $\omega = 2\pi f$ )

## Experiment #2: Compton Scattering (1923)



Classically, (initial)  $\lambda =$  (final)  $\lambda'$ . However, experiment shows  $\lambda \neq \lambda'$ !

Conservation of Energy and momentum in relativistic regime yields expression

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc} (1 - \cos\theta) \quad \text{independent of intensity, matching experimental results!}$$

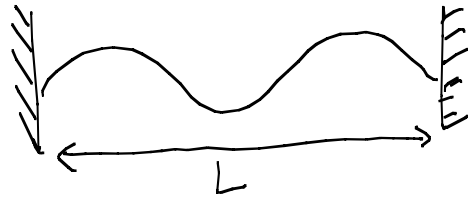
Conclusion: "light" is made up of particles ("photons") with discrete Energy  
 $E = hf$  and discrete momentum  $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$

So particle-wave duality is not unique to electrons!

Convincing evidence of wave nature of free electrons is provided by interference / diffraction (more on this later, including in-class demo ...)

but what about when electrons are confined e.g. in an atom?

⇒ When waves on a string are confined, discrete modes result.

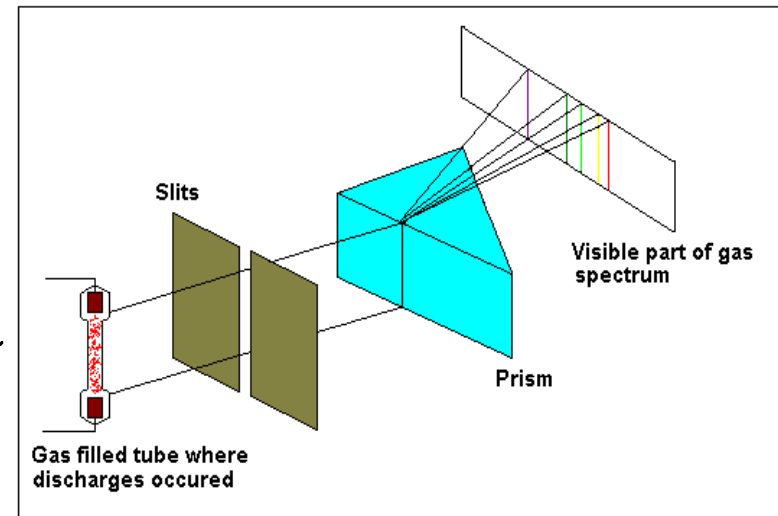


$$k_n = \frac{n\pi}{L}$$

Can we see the effects of discrete electron wave modes??

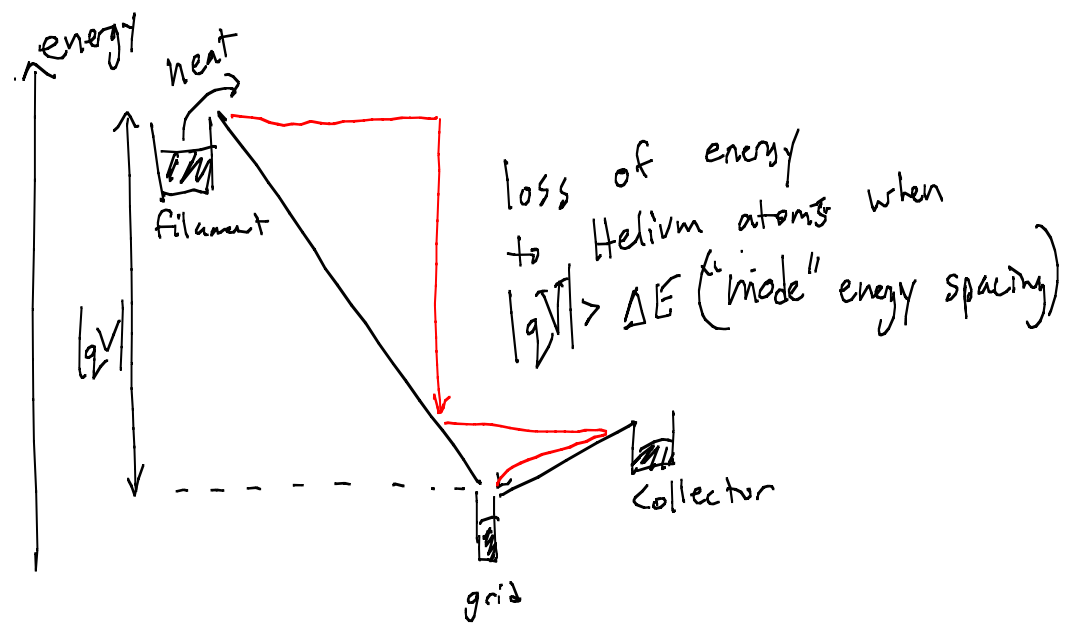
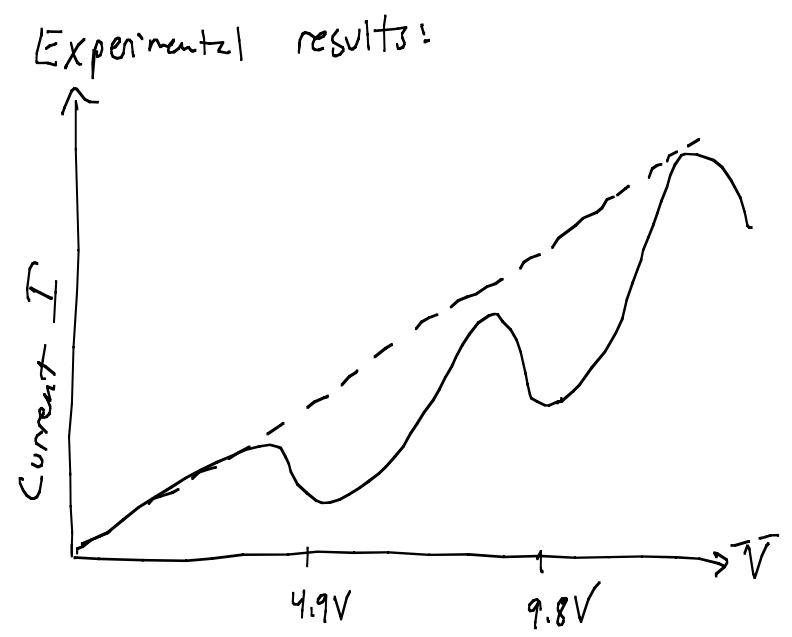
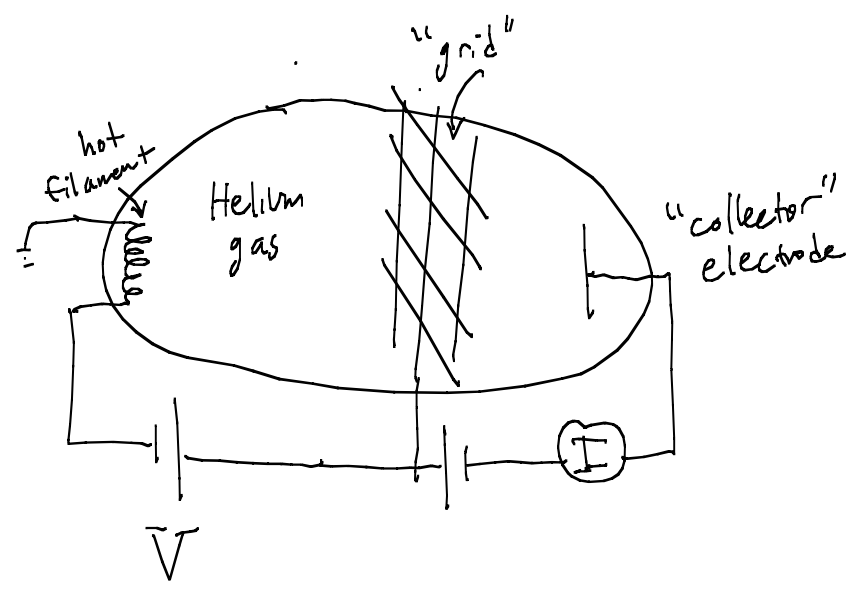
Yes! Atomic gases emit photons of well-defined wavelength (and hence energy)

when excited by collisions due to high voltage in discharge tubes: →



Discharges in the low-pressure gas filled tube are sources of light, which undergo refraction on a prism. We see the line spectrum of the gas.

# Further evidence of discrete electron wave "modes": Franck - G. Hertz (1914)



So electrons in Helium atom can only absorb discrete units of the free electron's kinetic energy, just like discrete modes of mechanical waves on a string!



If electron is a wave, then:

1. What is the wave equation it satisfies?

2. What is the corresponding dispersion relation?

3. What is the physical interpretation of this wave?

In this course, we will address these questions, then use their answers to solve basic problems in "quantum mechanics"!