1.1

```
Nmax=26;

N=zeros(1,Nmax); N(14)=1; N(15)=1; N(16)=3; N(22)=2; N(24)=2; N(25)=5;
P=N/sum(N); %normalized probability distribution
j=1:Nmax; %array of corresponding ages

%<j>
bjb=j*P';
disp('<j>^2=');
disp(bjb^2)

%<j^2>
bj2b=(j.^2)*P';
disp('<j^2>=')
disp(bj2b)

%standard deviation through sum of squares of differences
dj2=((j-bjb).^2)*P';
disp('delta j=')
disp(sqrt(dj2));

%standard deviation through expectation values
disp('compare with:')
disp(sqrt(bj2b-bjb^2))
```

$\langle j \rangle = \sum_j j P(j)$

$\langle j^2 \rangle = \sum_j j^2 P(j)$

$\sigma^2 = \sum (j - \langle j \rangle)^2 P(j)$

$\sigma^2 = \langle j^2 \rangle - \langle j \rangle^2$

Output:

```
<j>^2=
   441

<j^2>=
  459.5714

delta j=
    4.3095

compare with:
    4.3095
```

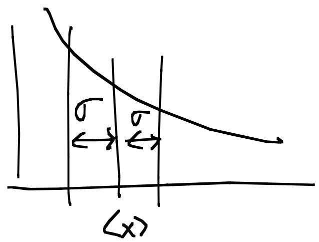
1.2

(a) $p(x) = \dfrac{1}{2\sqrt{hx}}$

$$\langle x^2 \rangle = \int_0^h \frac{x^2}{2\sqrt{hx}}\,dx = \frac{1}{2\sqrt{h}}\int_0^h x^{3/2}\,dx = \frac{1}{2h}\,\frac{2}{5}\,x^{5/2}\Big|_0^h = \frac{h^2}{5}$$

$$\text{so} \quad \sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{h^2}{5} - \left(\frac{h}{3}\right)^2} = \frac{2h}{3\sqrt{5}}$$

1.2 (b)



$$1 - \int_{\langle x\rangle-\sigma}^{\langle x\rangle+\sigma} \rho(x)\,dx = 1 - \int_{\langle x\rangle-\sigma}^{\langle x\rangle+\sigma} \frac{1}{2\sqrt{hx}}\,dx$$

$$= 1 - \frac{\sqrt{h/3 + \frac{2}{\sqrt{5}}\frac{h}{3}} - \sqrt{h/3 - \frac{2}{\sqrt{5}}h/3}}{\sqrt{h}}$$

$$= 1 - \frac{\sqrt{1+2/\sqrt{5}} - \sqrt{1-2/\sqrt{5}}}{\sqrt{3}} \sim 0.39$$

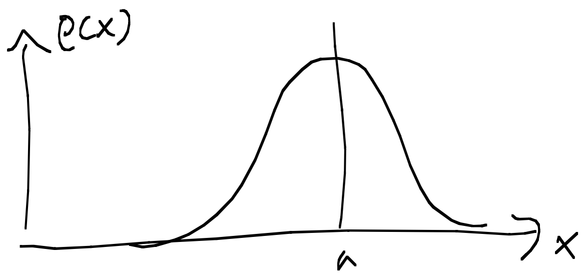
1.3 (a) $\int_{-\infty}^{\infty} \rho(x)\,dx = 1 \rightarrow \int A e^{-\lambda(x-a)^2}\,dx = A\sqrt{\frac{\pi}{\lambda}}$ so $A = \sqrt{\frac{\lambda}{\pi}}$

(b) by symmetry, $\langle x\rangle = a$

$\langle x^2\rangle = \sigma^2 + \langle x\rangle^2 = \frac{1}{2\lambda} + a^2$

$\frac{1}{2\sigma^2} = \lambda$ so $\sigma = \sqrt{\frac{1}{2\lambda}}$

(c)



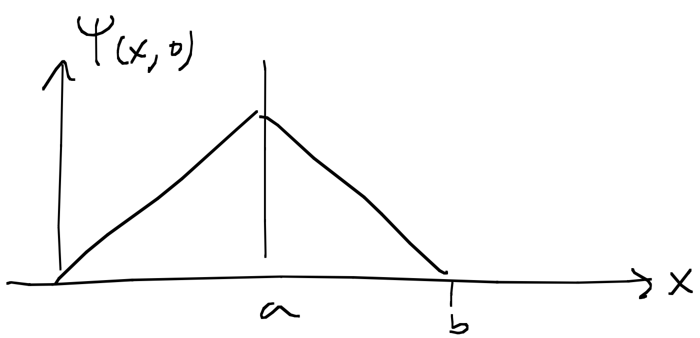
1.4

ⓐ $\int_{-\infty}^{\infty} \Psi^* \Psi \, dx = 1 \longrightarrow \int_0^a A^2 \frac{x^2}{a^2} dx + \int_a^b A^2 \frac{(b-x)^2}{(b-a)^2} dx = 1$

$$= A^2 \left[\frac{x^3}{3a^2} \Big|_0^a - \frac{(b-x)^3}{3(b-a)^2} \Big|_a^b \right] = A^2 \left[\frac{a}{3} - \left(- \frac{b-a}{3} \right) \right]$$

$$= A^2 \frac{b}{3} \quad \text{so} \quad A = \sqrt{\frac{3}{b}}$$

ⓑ



ⓒ $\max(\Psi(x,0)) \rightarrow x = a$

ⓓ $\int_0^a \Psi^* \Psi \, dx = \int_0^a A^2 \frac{x^2}{a^2} dx = A^2 \frac{x^3}{3a^2} \Big|_0^a = A^2 \frac{a}{3} = \frac{a}{b}$

(e) $\langle x \rangle = \int \Psi^* x \Psi \, dx = A^2 \left(\int_0^a \frac{x^3}{a^2} dx + \int_a^b \frac{x(b-x)^2}{(b-a)^2} dx \right)$

$$= \frac{3}{b} \left(\frac{x^4}{4a^2} \Big|_0^a + \frac{b^2 x^2/2 - 2b x^3/3 + x^4/4}{(b-a)^2} \Big|_a^b \right)$$

$$= \frac{3}{4b(b-a)^2} \left(a^2 (b-a)^2 + 2b^4 - 8\frac{b^4}{3} + b^4 - 2a^2b^2 + 8a^3b/3 - a^4 \right)$$

$$= \frac{3}{4b(b-a)^2} \left(\frac{b^4}{3} - a^2b^2 + \frac{2}{3}a^3 b \right) = \frac{(2a+b)(b-a)^2}{4(b-a)^2} = \frac{2a+b}{4}$$

1.5

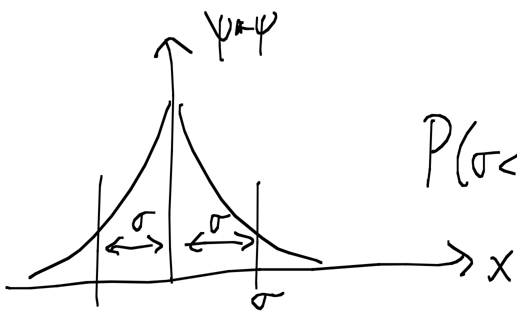
(a) $\int_{-\infty}^{\infty} \Psi^* \Psi \, dx = 2 \int_0^{\infty} A^2 e^{-2\lambda x} dx = 2A^2 \left(-\frac{e^{-2\lambda x}}{2\lambda} \right) \Big|_0^{\infty} = \frac{A^2}{\lambda} = 1$ so $A = \sqrt{\lambda}$

(b) $\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi^* x^2 \Psi \, dx = 2\lambda \int_0^{\infty} x^2 e^{-2\lambda x} dx = 2\lambda \int_0^{\infty} \frac{d^2}{d\xi^2} e^{-\xi x} dx \quad (\xi = 2\lambda)$

$$= 2\lambda \frac{d^2}{d\xi^2} \int_0^{\infty} e^{-\xi x} dx = 2\lambda \frac{d^2}{d\xi^2} \frac{1}{\xi} = 2\lambda \frac{2}{\xi^3} = \frac{4\lambda}{8\lambda^3} = \frac{1}{2\lambda^2}$$

$\langle x \rangle = 0$ by symmetry

(c) $\sigma = \sqrt{\langle x^2 \rangle} = \sqrt{\frac{1}{2\lambda^2}} = \frac{1}{\sqrt{2}\lambda}$



$$P(\sigma < x < \infty) = 2\lambda \int_\sigma^\infty e^{-2\lambda x}\, dx = 2\lambda \left(-\frac{e^{-2\lambda x}}{2\lambda} \right) \Bigg|_\sigma^\infty = e^{-2\lambda\sigma} = e^{-\sqrt{2}}$$

P. 1.17

(a)

$$1 = |A|^2 \int_{-a}^{a} (a^2 - x^2)^2 dx = 2|A|^2 \int_0^a (a^4 - 2a^2x^2 + x^4) dx = 2|A|^2 \left[a^4 x - 2a^2 \frac{x^3}{3} + \frac{x^5}{5} \right] \Big|_0^a$$

$$= 2|A|^2 a^5 \left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{16}{15} a^5 |A|^2, \text{ so } \boxed{A = \sqrt{\frac{15}{16a^5}}.}$$

(b) $\langle x \rangle = \int_{-a}^{a} x |\Psi|^2 \, dx = \boxed{0.}$ (Odd integrand.)

(c)

$$\langle p \rangle = \frac{\hbar}{i} A^2 \int_{-a}^{a} (a^2 - x^2) \underbrace{\frac{d}{dx}(a^2 - x^2)}_{-2x} dx = \boxed{0.} \quad \text{(Odd integrand.)}$$

Since we only know $\langle x \rangle$ at $t = 0$ we cannot calculate $d\langle x \rangle / dt$ directly.

(d)

$$\langle x^2 \rangle = A^2 \int_{-a}^{a} x^2 (a^2 - x^2)^2 dx = 2A^2 \int_0^a (a^4 x^2 - 2a^2 x^4 + x^6) dx$$

$$= 2 \frac{15}{16a^5} \left[a^4 \frac{x^3}{3} - 2a^2 \frac{x^5}{5} + \frac{x^7}{7} \right] \Big|_0^a = \frac{15}{8a^5} (a^7) \left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7} \right)$$

$$= \frac{\cancel{15} a^2}{8} \left(\frac{35 - 42 + 15}{\cancel{3} \cdot \cancel{5} \cdot 7} \right) = \frac{a^2}{8} \cdot \frac{8}{7} = \boxed{\frac{a^2}{7}.}$$

(e)

$$\langle p^2 \rangle = -A^2 \hbar^2 \int_{-a}^{a} (a^2 - x^2) \underbrace{\frac{d^2}{dx^2}(a^2 - x^2)}_{-2} dx = 2A^2 \hbar^2 2 \int_0^a (a^2 - x^2) dx$$

$$= 4 \cdot \frac{15}{16a^5} \hbar^2 \left(a^2 x - \frac{x^3}{3} \right) \Big|_0^a = \frac{15\hbar^2}{4a^5} \left(a^3 - \frac{a^3}{3} \right) = \frac{15\hbar^2}{4a^2} \cdot \frac{2}{3} = \boxed{\frac{5}{2} \frac{\hbar^2}{a^2}.}$$

(f)

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{1}{7} a^2} = \boxed{\frac{a}{\sqrt{7}}.}$$

(g) $\sigma_p = \sqrt{\langle p^2 \rangle - \cancel{\langle p \rangle^2}^{\,0}} = \sqrt{\frac{5}{2} \frac{\hbar^2}{a^2}} = \sqrt{\frac{5}{2}} \frac{\hbar}{a}$

(h) $\sigma_p \cdot \sigma_x = \sqrt{\frac{5}{2}} \frac{\hbar}{a} \frac{a}{\sqrt{7}} = \sqrt{\frac{5}{14}} \hbar = \sqrt{\frac{10}{7}} \frac{\hbar}{2} > \frac{\hbar}{2}$