Physics 401 - Homework #9

1) **Time evolution of expectation values (three points each).** If an observable (A) has a quantum mechanical operator ($\hat{A}$) which does not depend on time, then the time-rate change of the expectation value of that observable in any arbitrary state is given by

$$\frac{d\langle A\rangle}{dt} = \left\langle \frac{i}{\hbar}\left[\hat{H}, \hat{A}\right]\right\rangle$$

a) Apply the result shown above to the case where the observable (A) is the momentum. Let the Hamiltonian for the system be that of a particle moving in an arbitrary one-dimensional potential function V(x) . What famous law of classical physics does this result correspond to?

b) If the expectation value for an observable is constant in time, we say that that observable is conserved. Suppose we have a free particle (V(x) = constant). Is momentum conserved? Is energy conserved?

c)) Suppose we have a non-free particle (V(x) ≠ constant). Is momentum conserved? Is energy conserved?

2) **Orbital angular momentum operators (three points).** The quantum mechanical operators for orbital angular momentum are defined by analogy with the classical angular momentum:

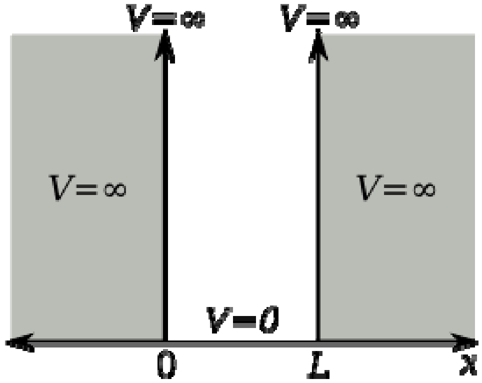
$$\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$$
$$\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z$$
$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$$

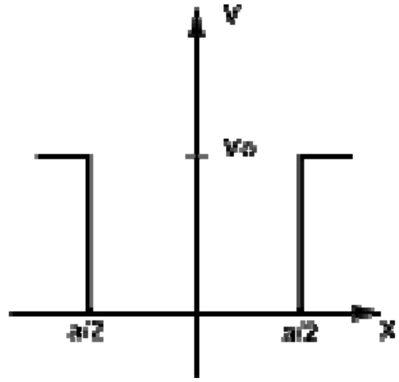
Using the "canonical commutation relations":

$$\left[\hat{x}, \hat{p}_x\right] = i\hbar$$
$$\left[\hat{y}, \hat{p}_y\right] = i\hbar\ ,$$
$$\left[\hat{z}, \hat{p}_z\right] = i\hbar$$

calculate the commutator of L_x and L_y. Hint #1: there is no need to use the explicit form of these operators to answer this question. The canonical commutation relations alone are enough to determine the result. Hint #2: operators for orthogonal directions commute. So, for example, the commutator of $\hat{x}$ and $\hat{p}_y$ is zero.

3) **The wavefunctions of energy eigenstates.** (three points each). Consider the infinite square well and the finite square well:





Sketch the spatial wavefunctions for the following energy eigenstates:

a) the ground state of the infinite square well.

b) the first excited state of the infinite square well.

c) the ground state of the finite square well.

d) the first excited state of the finite square well.

e) Inspect your sketches of the wavefunctions for the ground states of the infinite square well and the finite square well. If the widths of the wells are the same, does the finite square well have a lower energy ground state or a higher energy ground state? To answer this question, you do not need to do any calculation. Just look at the wavefunctions are make an argument about which has a lower energy.