Physics 401 - Homework #1

1) (3 points) Complex numbers are commonly expressed in two forms: cartesian ($z = x + iy$) and polar ($z = Ae^{i\vartheta}$). Convert $\frac{1}{1-i}$ to both of these forms, and state explicitly the values of x, y, A, and θ.

2) For any complex number, $\left|\frac{z^*}{z}\right| = 1$.

a) (3 points) Show that this is true using the cartesian form ($z = x + iy$).

b) (3 points) Show that this is true using the polar form ($z = Ae^{i\vartheta}$).

Which method is easier?

3) (1 point each, 4 points total) Prove the following identities for complex numbers:

a) Re(z) = (z +z*)/2
b) Im(z) = (z - z*)/2i
c) cos(θ) = [exp(iθ) + exp(-iθ)]/2
d) sin(θ) = [exp(iθ) - exp(-iθ)]/2i

4) (1 point each, 4 points total)

a) What is the phase of the following wavefunction?:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

b) Show that the phase is shifted by π/2 when the function is multiplied by i
c) Show that the phase is shifted by π when the function is multiplied by -1.
d) Suppose we multiply the wavefunction by a phase factor $e^{i\alpha}$. Show that this does not change the value of $|\Psi(x,t)|^2$.

5) **Expectation value of a discrete variable.** (three points each, 18 points total). The "quantum" in quantum mechanics refers to the fact that the energy of bound states is discrete, or quantized, in microscopic systems. Suppose that we have 15 identical quantum systems, and we precisely measure the energy of each one. We find the following results, measured in units of electron-Volts (eV):

$$\{E_i\} = \{8,5,4,5,4,6,7,5,6,4,4,5,6,7,4\}$$

a) Draw a histogram of these results.
b) What was the probability of getting each of the five energy values?
c) What was the most probable value?
d) Calculate the expectation value of the energy of this system.
e) Calculate the expectation value of the square of the energy of this system.
f) Calculate the variance and standard deviation of the energy of this system.

6) **Expectation value of a continuous variable.** (3 points each, 18 points total). Energy is not always quantized, even in quantum mechanics. For example, free particles can have a continuum of energies. Suppose we have a very large number of identical free particles, and after measuring the energies of all of them, we find that the probability distribution of the energy in electron-Volts is described by

$$P(E) = \begin{Bmatrix} 0, E < 3 \\ \alpha(E-3), 3 \le E \le 8 \\ 0, E > 8 \end{Bmatrix}$$

where (α) is a normalization constant.

a) Calculate the normalization constant (α). What are its units?
b) Sketch the normalized probability distribution.
c) What is the probability of measuring an energy between 7.0 eV and 7.1 eV?
d) Calculate the expectation value of the energy of this system.
e) Calculate the expectation value of the square of the energy for this system.
f) Calculate the variance and standard deviation of the energy of this system.