Phys 401 Final Exam Formula Sheet

$$\psi''(x) = \frac{-2m}{\hbar^2}(E - V(x))\psi(x)$$

$$J \equiv \frac{\hbar}{2mi}\left(\psi^*\psi' - \psi\left(\psi^*\right)'\right),\ T \equiv \frac{\left|J_{trans}\right|}{\left|J_{inc}\right|}, R \equiv \frac{\left|J_{refl}\right|}{\left|J_{inc}\right|}$$

$$[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z,\ [\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x,\ [\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y$$

$$[\hat{L}_x, \hat{L}^2] = 0,\ [\hat{L}_y, \hat{L}^2] = 0,\ [\hat{L}_z, \hat{L}^2] = 0$$

$$\hat{L}_\pm \equiv \hat{L}_x \pm i\hat{L}_y,\ [\hat{L}_z, \hat{L}_+] = \hbar\hat{L}_+,\ [\hat{L}_z, \hat{L}_-] = -\hbar\hat{L}_-,\ [\hat{L}^2, \hat{L}_\pm] = 0,\ [\hat{L}_+, \hat{L}_-] = 2\hbar\hat{L}_z$$

$$\hat{L}^2\left|lm\right\rangle = \hbar^2 l(l+1)\left|lm\right\rangle,\ l = 0,1,2,3,\ldots$$

$$\hat{L}_z\left|lm\right\rangle = m\hbar\left|lm\right\rangle,\ m = -l,\ldots,+l$$

$$\hat{L}_\pm\left|lm\right\rangle = \hbar\sqrt{l(l+1) - m(m\pm1)}\left|l, m\pm1\right\rangle$$

$$V(\vec{r}) = V(r),\ \hat{H} = \frac{-\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{\hbar^2 l(l+1)}{2mr^2} + V(r),\ \phi(r,\theta,\varphi) = R(r)Y_l^m(\theta,\varphi)$$

$$u(r) \equiv rR(r),\ \frac{-\hbar^2}{2m}\frac{d^2u(r)}{dr^2} + \left[V(r) + \frac{\hbar^2 l(l+1)}{2mr^2}\right]u(r) = Eu(r)$$

$$V(r) = \frac{-kZe^2}{r},\quad u(r) = \rho^{\ell+1}e^{-\rho}\left[\sum_{i=0}^{\infty} c_j\rho^j\right]$$

$$\rho \equiv \kappa r,\ \kappa \equiv \frac{\sqrt{-2mE}}{\hbar} = \frac{1}{a_0 n},\ a_0 \equiv \frac{\hbar^2}{mkZe^2} = 0.0529\,\text{nm (for Z = 1)}$$

$$c_{j+1} = \left[\frac{2\left(j+\ell+1-n\right)}{\left(j+1\right)\left(j+2\ell+2\right)}\right]c_j,\ n = 1,2,3,\ldots$$

$$E_n = \frac{-mk^2Z^2e^4}{2\hbar^2}\frac{1}{n^2} = \frac{-13.6eV}{n^2}\ \text{(for Z = 1)}$$

$$\ell \le n-1$$

$$\phi_{100}(r,\varphi,\theta) = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$$