Physics 401 - Homework #1

1) (4 points total) Convert the following complex numbers into the $z = x + iy$ **and** $z = Ae^{i\vartheta}$ forms. State explicitly the values for x, y, A, and θ for each.

a) i^i

b) $\frac{1}{1-i}$

2) (4 points total) Prove the following identities for complex numbers:

a) Re(z) = (z +z*)/2

b) Im(z) = (z - z*)/2i

c) cos(θ) = [exp(iθ) + exp(-iθ)]/2

d) sin(θ) = [exp(iθ) - exp(-iθ)]/2i

3) (3 points total)

a) Write down the phase of the following wavefunction:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

b) Show that the phase is shifted by π/2 when the function is multiplied by i

c) Show that the phase is shifted by π when multiplied by -1.

4) (3 pts) Complex numbers are useful for quickly deriving trigonometric identities. Prove the laws of addition of sines and cosines

$$\sin(\theta+\varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi$$
$$\cos(\theta+\varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$$

using Euler's formula ($\exp(i\theta) = \cos\theta + i\sin\theta$) and exponential notation. Hint: work out $\exp(i(\theta+\varphi))$, and then take the real and imaginary parts.

5) **Classical harmonic oscillator**. (5 points total) A particle of mass (m) is attached to a spring. The spring exerts a force on the mass proportional to the displacement from equilibrium, and the force acts opposite to the displacement (it is a restoring force). Therefore the force is described by F = -kx, where (k) is the spring constant, and (x) is the displacement measured from the equilibrium position.

a) Write down Newton's second law of motion for this system as a differential equation. The equation should be satisfied by a function x(t) which describes the particle's position as a function of time..

b) Find a general solution to the equation.

c) What is the angular frequency of the oscillator in terms of (k) and (m)?

d) How many initial conditions are necessary to specify a particular solution?

e) Choose a set of initial conditions and find the corresponding particular solution. Please don't choose the trivial solution: $x(t) = 0$!.