Phys 401 Final Exam Formula Sheet

$$i\hbar \frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$$

$$i\hbar \frac{d}{dt} c_i(t) = \sum_j H_{ij} c_j(t), \quad c_i \equiv \langle i|\psi\rangle_t, \quad H_{ij} \equiv \langle i|\hat{H}|j\rangle$$

$$\left(A^t\right)^*_{ij} \equiv A_{ji}$$

$$|\psi'\rangle = U|\psi\rangle,\ F' = UFU^{-1},\ U_{mi} \equiv \langle m|i\rangle,\ U^{-1} = U^t$$

$$\hat{H} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix},\ |up\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ |down\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\quad \begin{aligned} i\hbar \frac{d}{dt} c_1(t) &= E_0 c_1(t) - A c_2(t) \\ i\hbar \frac{d}{dt} c_2(t) &= -A c_1(t) + E_0 c_2(t) \end{aligned}$$

$$\begin{aligned} |I\rangle &= \frac{1}{\sqrt{2}}\left(|up\rangle + |down\rangle\right) \\ |II\rangle &= \frac{1}{\sqrt{2}}\left(|up\rangle - |down\rangle\right) \end{aligned},\quad \begin{aligned} E_I &= E_0 - A \\ E_{II} &= E_0 + A \end{aligned}$$

if $|\psi(t=0)\rangle = |up\rangle$, then $|\psi(t)\rangle = e^{-iE_0 t/\hbar}\left[\cos\left(\frac{At}{\hbar}\right)|up\rangle + i\sin\left(\frac{At}{\hbar}\right)|down\rangle\right]$

$$\psi''(x) = \frac{-2m}{\hbar^2}(E - V(x))\psi(x)$$

$$J \equiv \frac{\hbar}{2mi}\left(\psi^*\psi' - \psi\left(\psi^*\right)'\right),\ T \equiv \frac{|J_{trans}|}{|J_{inc}|}, R \equiv \frac{|J_{refl}|}{|J_{inc}|}$$

$$[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z,\ [\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x,\ [\hat{L}_z, \hat{L}_x] = i\hbar\hat{L}_y$$

$$[\hat{L}_x, \hat{L}^2] = 0,\ [\hat{L}_y, \hat{L}^2] = 0,\ [\hat{L}_z, \hat{L}^2] = 0$$

$$\hat{L}_\pm \equiv \hat{L}_x \pm i\hat{L}_y,\ [\hat{L}_z, \hat{L}_+] = \hbar\hat{L}_+,\ [\hat{L}_z, \hat{L}_-] = -\hbar\hat{L}_-,\ [\hat{L}^2, \hat{L}_\pm] = 0,\ [\hat{L}_+, \hat{L}_-] = 2\hbar\hat{L}_z$$

$$\hat{L}^2|lm\rangle = \hbar^2 l(l+1)|lm\rangle,\ l = 0,1,2,3,\ldots$$

$$\hat{L}_z|lm\rangle = m\hbar|lm\rangle,\ m = -l,\ldots,+l$$

$$\hat{L}_\pm|lm\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}\,|l, m\pm 1\rangle$$

$$\langle \theta\varphi | lm \rangle = Y_l^m(\theta,\varphi) = e^{im\varphi}(-1)^m \sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}} P_l^m(\cos\theta) \, ,$$

$$P_l^m(x) \equiv (1-x^2)^{|m|/2} \left(\frac{d}{dx}\right)^{|m|} P_l(x) \, , \; P_l(x) = \frac{1}{2^l l!} \left(\frac{d}{dx}\right)^l (x^2-1)^l$$

$$\langle l'm' | lm \rangle = \delta_{l'l} \delta_{m'm} \, ,$$

$$\int_{4\pi} \left(Y_{l'}^{m'}\right)^* \left(Y_l^m\right) d\Omega = \int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\varphi \left(Y_{l'}^{m'}\right)^* \left(Y_l^m\right) = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\varphi \left(Y_{l'}^{m'}\right)^* \left(Y_l^m\right) = \delta_{l'l} \delta_{m'm}$$

$$d\Omega = d(\cos\theta) d\varphi = \sin\theta d\theta d\varphi$$

$$\sum_{l=0}^{\infty} \sum_{m=-l}^{l} |lm\rangle\langle lm| = 1 \, , \; \psi(\theta,\varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta,\varphi) \, , \; a_{lm} = \langle lm | \psi \rangle = \int_{4\pi} \left(Y_l^m\right)^* \psi d\Omega$$

$$V(\vec{r}) = V(r) \, , \; \hat{H} = \frac{-\hbar^2}{2m} \frac{1}{r} \frac{\partial^2}{\partial r^2} r + \frac{\hbar^2 l(l+1)}{2mr^2} + V(r) \, , \; \varphi(r,\theta,\varphi) = R(r) Y_l^m(\theta,\varphi)$$

$$u(r) \equiv rR(r) \, , \; \frac{-\hbar^2}{2m} \frac{d^2 u(r)}{dr^2} + \left[V(r) + \frac{\hbar^2 l(l+1)}{2mr^2} \right] u(r) = Eu(r)$$