Phys 401 Exam #2 Formula Sheet

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik(x-x')}dk = \delta(x-x'), \quad \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(k-k')x}dx = \delta(k-k')$$

$$\langle m|n\rangle = \delta_{mn}, \qquad \langle k|k'\rangle = \delta(k-k'), \qquad \langle x|x'\rangle = \delta(x-x')$$

$$\sum_n |n\rangle\langle n| = \hat{I}, \qquad \int |k\rangle\langle k|dk = \hat{I}, \qquad \int |x\rangle\langle x|dx = \hat{I}$$

$$\langle n|\psi\rangle = a_n, \qquad \langle k|\psi\rangle = \phi(k), \qquad \langle x|\psi\rangle = \psi(x)$$

$$|\psi\rangle = \sum_n a_n|n\rangle, \quad |\psi\rangle = \int \phi(k)|k\rangle dk, \qquad |\psi\rangle = \int \psi(x)|x\rangle dx$$

$$\langle x|n\rangle = \varphi_n(x), \quad \langle x|k\rangle = \frac{1}{\sqrt{2\pi}}e^{ikx}, \qquad \langle x|x'\rangle = \delta(x-x')$$

$$\langle C\rangle = \langle\psi|\hat{C}|\psi\rangle$$

$$\langle \hat{A}^t\alpha|\beta\rangle \equiv \langle\alpha|\hat{A}\beta\rangle$$

$$\Delta A\Delta B \geq \frac{1}{2}\left|\langle C\rangle\right|, \quad \hat{C} \equiv [\hat{A},\hat{B}]$$

$$\frac{d\langle A\rangle}{dt} = \left\langle \frac{i}{\hbar}\left[\hat{H},\hat{A}\right]\right\rangle$$

$$V(x) = \frac{1}{2}Kx^2, \ \omega_0 = \sqrt{\frac{K}{m}}, \ \beta \equiv \sqrt{\frac{m\omega_0}{\hbar}},$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}Kx^2, \quad \varphi_0(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} e^{-(\beta x)^2/2}$$

$$\hat{a} \equiv \frac{\beta}{\sqrt{2}}\left(\hat{x} + \frac{i\hat{p}}{m\omega_0}\right), \quad \hat{a}^t \equiv \frac{\beta}{\sqrt{2}}\left(\hat{x} - \frac{i\hat{p}}{m\omega_0}\right), \ \hat{N} \equiv \hat{a}^t\hat{a}, \ \hat{H} = \hbar\omega_0\left(\hat{N} + \frac{1}{2}\right)$$

$$\hat{H}|n\rangle = E_n|n\rangle, \ E_n = \hbar\omega_0\left(n + \frac{1}{2}\right), \ \hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \ \hat{a}^t|n\rangle = \sqrt{n+1}|n+1\rangle,$$

$$\xi \equiv \beta x, \ \varphi_n(\xi) = A_n\left(\xi - \frac{d}{d\xi}\right)^n e^{-\xi^2/2} \equiv A_n H_n(\xi)e^{-\xi^2/2}, \ A_n = \left(2^n n!\sqrt{\pi}\right)^{-1/2}$$