Physics 401 - Homework #4 - Due Wednesday October 7th

1) **Incompatible observables (one point each)**. For this question, you do not need to do any calculations, except a very brief one for part (c). For all others, just explain your reasoning.

In quantum mechanics, if the measurement of an observable A disturbs observable B, we say that A and B are incompatible observables. For example, suppose we have a quantum mechanical free particle in a particular state $\psi(x)$, and we make the following measurements.

a) We perform a position measurement of the particle with an **extremely** accurate device, and the result is $x = x^*$. What is the state of the particle immediately after this measurement? (In other words, what is its wavefunction after the measurement?)

b) What is the uncertainty in the position of the particle, immediately after making the position measurement?

c) Calculate the momentum-space wavefunction of the particle immediately after the measurement.

d) What is the uncertainty in the momentum of the particle immediately after the position measurement?

e) We follow up with a measurement of the momentum of the particle, with **extremely** high accurately, and the result is $p = p^*$. What will be the uncertainty in the position of the particle after this momentum measurement?

2) **Compatible observables (one point each).** (Again, no calculations are needed for this question, just explain.)

Suppose we have a plane wave free particle, with wavenumber k^*.

a) What is the uncertainty in the momentum of the particle?

b) What is the uncertainty in the energy of the particle?

c) We measure the energy of the particle. Is the momentum of the particle disturbed?

d) We measure the energy, then the momentum, then the energy again. What is the result of this final energy measurement?

e) Do these observables exhibit an uncertainty principle?

3) **Commutators and compatibility (six points total)**.

a) Because position and momentum are incompatible observables in quantum mechanics, they cannot have any eigenfunctions in common. Prove this by assuming that there exists a state $\psi(x)$ which is an eigenfunction of both x and p, with eigenvalues x^* and p^*. Show that this assumption leads to a contradiction of the position-momentum commutator which you calculated in Homework #2. Hint: you do not need to use the explicit forms of the x and p operators in your proof, just use the eigenvalue equations instead.

b) Generalize your result from part (a) for any operators A and B. Show that if A and B have just one eigenfunction in common, then their commutator must be zero.

4) **Triangle wavefunction in a box (9 points).** Suppose a particle in a box has a triangular initial state wavefunction:

$$\psi(x) = \begin{cases} A\left(x + \frac{L}{2}\right), & -L/2 < x < 0 \\ A\left(\frac{L}{2} - x\right), & 0 < x < L/2 \\ 0, & otherwise \end{cases}$$

As you can see from the wavefunction, we have chosen to measure the (x) coordinate from the center of the box for this problem. This makes the mathematics simpler. In this coordinate system the stationary states appear as

$$\varphi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\cos\left(\frac{n\pi x}{L}\right), & n = odd \\ \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right), & n = even \end{cases}$$

a) What are the units of the normalization constant (A) (one point)?

b) Calculate (A) in terms of (L) (three points).

c) Expand the wavefunction $\psi(x)$ in terms of the stationary states, and calculate the expansion coefficients $\{a_n\}$. Hint: the integral for $\{a_n\}$ should be done for all (n), both even and odd. However, in the case of even (n), the integral is zero, because the integrand turns out to be an odd function evaluated over a symmetric interval. So you only need to deal with the case of odd (n). (three points)

d) If the particle is an electron, and it is trapped in a conductor which is one nanometer long, calculate the probability that an energy measurement leaves the electron in the n = 3 state, and calculate the energy of that state in Joules. (one point).

e) Is there any limit to the energy which might be obtained in a single energy measurement? (one point).