Physics 401 - Homework #12 - Due Wednesday December 9th

1) **A spherical harmonic (three points)**. Apply the definition of the spherical harmonics to calculate the explicit functional form for $Y_3^1(\theta,\phi)$. The definition is given in Griffiths, equations 4.27, 4.28, and 4.32.

2) **L_x eigenstates**. L_x and L_z do not have eigenstates in common, because their operators do not commute. However, we can find eigenstates of L_x which are linear combinations of the eigenstates of L_z.

a) (one point) Determine an expression for the (L_x) operator in terms of the ladder operators (L_+) and (L_-) (L-plus and L-minus).

b) (three points) Show that the following combinations of angular momentum states are eigenstates of L_x, and identify their eigenvalues. Hint: use the form of the L_x operator determined in part (a).

$$\left|Lx-state-1\right\rangle = \frac{1}{2}\left(\left|1,1\right\rangle - \sqrt{2}\left|1,0\right\rangle + \left|1,-1\right\rangle\right)$$

$$\left|Lx-state-2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|1,1\right\rangle - \left|1,-1\right\rangle\right)$$

$$\left|Lx-state-3\right\rangle = \frac{1}{2}\left(\left|1,1\right\rangle + \sqrt{2}\left|1,0\right\rangle + \left|1,-1\right\rangle\right)$$

c) (two points) Suppose that a particle is in the eigenstate "Lx-state-2", defined in part (b), and we measure the z component of the angular momentum. What values might we observe, and what are the probabilities for each of those values?

3) **A rigid rotator.** Suppose that a rigid rotator has an angular wavefunction of

$$\psi(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\sin(\theta)\sin(\varphi)$$

a) (three points) What are the possible values of L^2 and Lz that might be observed for this rotator? Hint: you should expand this wavefunction in terms of the spherical harmonics. You can probably guess the correct expansion by inspecting some of the lower level spherical harmonic functions.

b) (three points) Suppose that we allow the wavefunction of evolve in time. Write down an expression for the time-dependent wavefunction. Hint: the Hamiltonian for this system is

$$\hat{H} = \frac{\hat{L}^2}{2I}$$

where I is the moment of inertia of the rotator. Your answer will depend on I.

c) (three points) Suppose we measure the position of the particle, at t = 0. What is the probability that the particle will be observed with a theta between zero degrees and 30 degrees?

4) **Angular momentum operator as a generator (three points).** Show that the Lz operator generates angular rotations about the z-axis:

$$\exp(i\Delta\varphi\hat{L}_z / \hbar) f(\varphi) = f(\varphi + \Delta\varphi)$$