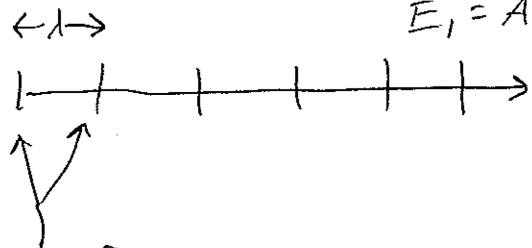
Interference of Light

Suppose we have two waves:

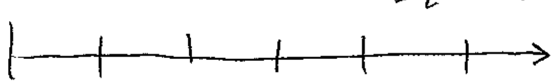
$E_1 = A_0 \sin \omega t$

waves overlap here



wavefronts

$E_2 = A_0 \sin(\omega t + \delta)$



In general the waves have some phase difference, which we call δ.

When the waves overlap we may observe interference.

To see interference we need

- same polarization
- coherent waves

} often we get this by using the same light source for both waves.

Assume polarization is the same, so we can ignore the vector nature of light. Then

$$E_p = E_1 + E_2 = A_0 \left[\sin \omega t + \sin(\omega t + \delta) \right]$$

Trig identity: $\sin \alpha + \sin \beta = 2 \cos\left(\frac{\alpha - \beta}{2}\right) \sin\left(\frac{\alpha + \beta}{2}\right)$

Then $E = \underbrace{\left[2A_0 \cos \frac{\delta}{2} \right]}_{\text{"amplitude factor"}} \underbrace{\sin\left(\omega t + \frac{\delta}{2}\right)}_{\text{"wave factor"}}$

Wave factor is oscillating <u>really fast</u> $\rightarrow$ 10^{14} times per second for visible light

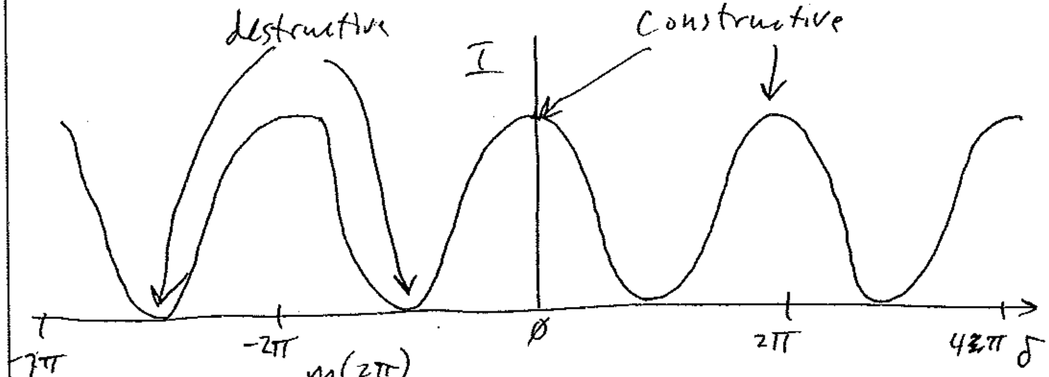
$\rightarrow$ your eye observes the average.

Intensity $I \propto E^2 = \left[4A_0^2\cos^2\frac{\delta}{2}\right] \cancel{\sin^2(\omega t+\delta)}$

unobserved in visible light

$$\boxed{I = 4A_0^2\cos^2\frac{\delta}{2}}$$

How the intensity depends on the phase difference.



When $\delta = m(2\pi)$, we get constructive interference

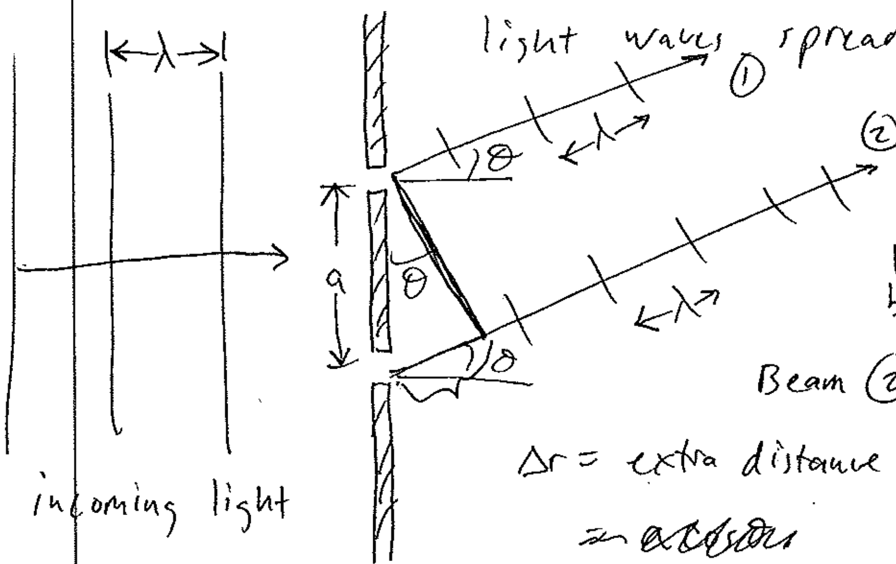
$m = 0, \pm 1, \pm 2, \pm 3, \ldots$

When $\delta = (m+\frac{1}{2})(2\pi)$, we get destructive interference.

<u>Example</u> Young's Double Slit experiment

Light illuminates a barrier with two narrow slits:

light waves spread out in all directions.



Here we consider one particular direction given by θ.

What's the phase difference between these two waves?

Beam ② travels a longer distance than ①

Δr = extra distance travelled by ②.

$= \cancel{a\cos\theta}$

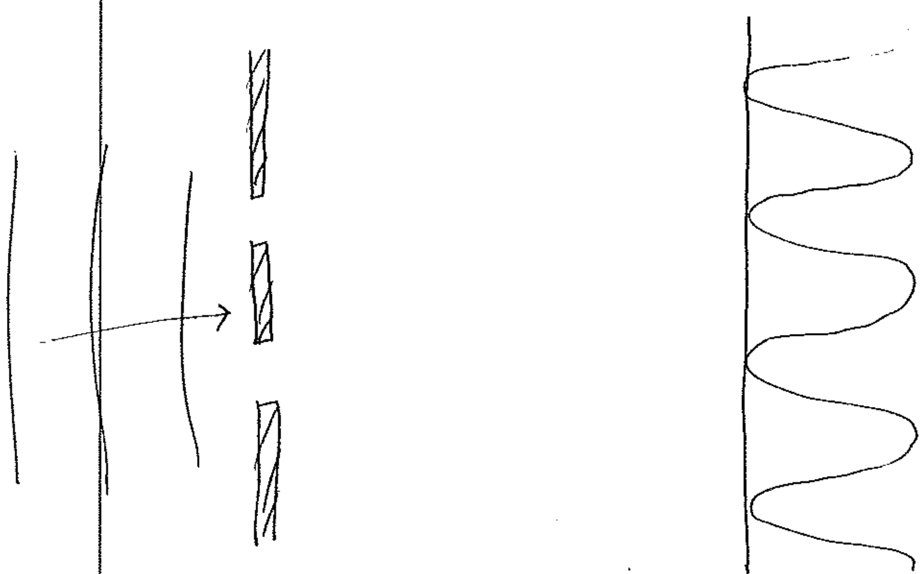
$= a\sin\theta$.

If $\Delta r = \lambda$, then the phase difference is $\underline{2\pi}$.

So $$\frac{\Delta r}{\lambda} = \frac{\delta}{2\pi}$$

$$\delta = \frac{2\pi}{\lambda}\Delta r = \frac{2\pi a \sin\theta}{\lambda}$$

Then $$I = 4A_0^2 \cos^2\frac{\delta}{2} = 4A_0^2 \cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$$



Intensity on a distant screen.

The key is the phase difference between the two paths. In general, a path length difference causes a phase difference

$$\frac{\Delta r}{\lambda} = \frac{\delta}{2\pi}$$

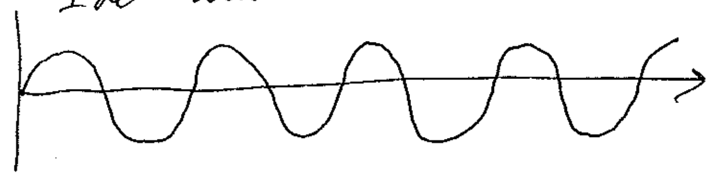
$$\delta = \underbrace{\frac{2\pi}{\lambda}}_{k}\Delta r = k\Delta r$$

$$\boxed{\delta = k\Delta r}$$ phase difference due to a path length difference Δr.

Coherence

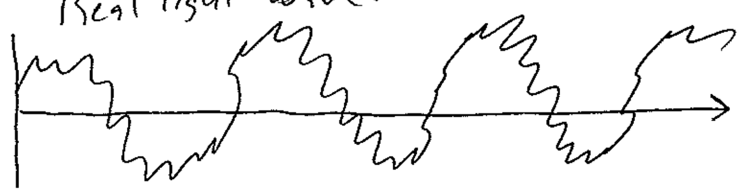
Real light waves aren't perfect sines & cosines.

Ideal wave



perfectly predictable for all time.

Real light wave.



Only predictable for a ~~some~~ finite time called the coherence time τ

Coherence length = coherence time × speed of light

$$\ell = \tau c$$

To observe interference, path length difference must be smaller than the coherence length.

For incandescent light, $\tau \sim$ one oscillation time

$\sim 10^{-14}$ sec. ← very incoherent

Then $\ell \sim 3\ \mu m$

For He-Ne lasers, $\tau \sim 1\ ns = 10^{-9}$ sec

$\ell \sim$ 30 cm.

The key feature of a laser that makes it different from conventional light is its coherence ⟹ much more like an ideal wave than incoherent conventional light

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