Phys 375 - Homework #1

1) (18 pts.) You measure the diameter of lens A with a ruler. The ruler is marked in one centimeter increments. You make the following set of measurements:

5.3, 5.1, 6.1, 5.6, 5.5, 5.9, 5.8, 5.5, 5.4, 5.3, 5.6, 4.9, 5.8, 5.7, 5.1, 5.6, 5.5 (cm)

When making these measurements, you have estimated by eye the last digit to the best of your ability.

a) Sketch a histogram of these measurements.
b) Calculate by hand (using a calculator rather than a computer) the mean, standard deviation, and standard deviation of the mean for this data. Please show your work.
c) What would you report as the measured value and uncertainty on the diameter of lens A, given the complete set of data that you have?
d) You now make a single measurement of the diameter of a different lens, lens B, with the same ruler and find a value of 7.8 cm. Your lab partner argues that the uncertainty on this measurement is 1 cm, because the ruler is marked in 1 cm increments. Do you agree or disagree? What would you report as the central value and uncertainty for the diameter of lens B based upon your single measurement?
e) Is your knowledge of the diameter of lens A better than, equal to, or worse than that of lens B? Why?
f) Suppose you need to know the diameter of lens B to a relative precision of 1%, and you only have the same ruler available to you for the measurement. How can that be done?

2) (12 pts.) You are trying to determine the acceleration due to gravity (g) using the following relationship between the period (T) of a pendulum and its length (L):

$$T = 2\pi\sqrt{\frac{L}{g}}$$

You measure T = 2.01 ± 0.30 seconds and L = 93.5 ± 4.0 cm.

a) Determine the uncertainty on (g) due to the uncertainty on (T) alone (ignoring the uncertainty in L) in two ways: using the traditional propagation-of-errors formula, and by explicitly varying (T) by its uncertainty and observing the resulting variation in (g). Is the uncertainty estimate the same for both methods or different? Why? Is the uncertainty for a positive variation the same as for a negative variation? Why?

b) Repeat part (a), this time for the uncertainty on (g) do to (L) alone.
c) Combine the uncertainties due to (T) and (L) together to determine the total uncertainty on (g). Which of the two sources of uncertainty is the more important one?

d) Which of the two methods for propagating errors produces a better estimate of the uncertainty on (g)? Which method is easier, and less prone to mistakes, in your opinion?

3) (3 pts.) Prove that for multiplicative formulas, the fractional error on the calculated value is equal to the fractional error for each factor added together in quadrature. For example, if f = abc, and a, b, and c have errors Δa, Δb, and Δc, the fractional error on f is given by

$$\left(\frac{\Delta f}{f}\right)^2 = \left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2 + \left(\frac{\Delta c}{c}\right)^2$$

4) (6 pts) You measure the positions of two neighboring diffraction maxima with a meter stick. The measurements are 56.7 ± 0.1 cm and 64.8 ± 0.1 cm.

a) What is the uncertainty in the distance between these maxima?
b) You find in your laboratory an exotic instrument: a kilometer stick (!!!). You decide to try it out, with the help of a large team of friends to move the kilometer stick around. After careful alignment, the zero point of the kilometer stick is halfway across campus next to the Testudo statue. You again measure the locations of the neighboring diffraction maxima in your lab, and you find positions of 93,917.2 ± 0.1 cm and 93,925.3 ± 0.1 cm. Has the fractional uncertainty on the location of each maxima changed by using the kilometer stick? Has the uncertainty on the distance between the maxima changed by using the kilometer stick? Explain your answers.

5) (9 pts) You measure the diameter of a lens to be 7.5 ± 0.3 cm.

a) What is the fractional uncertainty on the diameter?
b) Calculate the area of one side of the lens, assuming it to be a plane circle. What is the fractional uncertainty on the area? Is it larger, smaller, or the same as the fractional error on the diameter? Why?
c) You measure the diameter of a baseball. Coincidentally, it happens to be 7.5 ± 0.3 cm, just like the lens. Calculate the volume of the baseball. What is the fractional uncertainty on the volume? How does it compare to the fractional uncertainty on the diameter? How does it compare to the fractional uncertainty on the area of the lens from part (b)? Why?