Measurement of the index of refraction of a glass block

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Abstract

This is an example lab report for part 1 of Lab 1. The report emphasizes how the data was analyzed, and how the uncertainties were determined and propagated. Systematic uncertainties and possible methods to improve the measurement are also discussed. Note that this report only covers part 1 of the lab, although you are responsible for reporting on parts 2 and 3 as well.

1. Introduction

I performed two measurements of the index of refraction of a glass block, once measuring the laser beam displacement with a ruler, and once measuring with the photodiode and stepper motor. For both measurements the glass block was oriented vertically, so the beam passed through its shortest dimension.

2. Uncertainties on the directly measured quantities

2.1. Angle of incidence

I measured the incident angle with the vernier scale on the rotation table, using retro-reflection to determine the direction normal to the glass surface. I repeated the retro-reflection procedure four times and found that the reading varied by 0.1°, so I adopt this as the uncertainty on the normal direction, as well as the uncertainty on all the other angles measured in this portion of the lab. The angle of incidence for each measurement is the difference between the rotated reading and the normal direction, so the uncertainty on the normal direction and the uncertainty on the rotated value add in quadrature:

$$\sigma_{\theta_i} = \sqrt{(0.1^2 + 0.1^2)} = 0.14^\circ \tag{1}$$

2.2. Glass thickness

I measured the thickness of the glass block several times and in several different locations with the calipers, and each measurement was 9.15 mm. Since I found no variation from one measurement to the next, I adopt the smallest digit on the calipers, 0.05 mm, as the uncertainty.

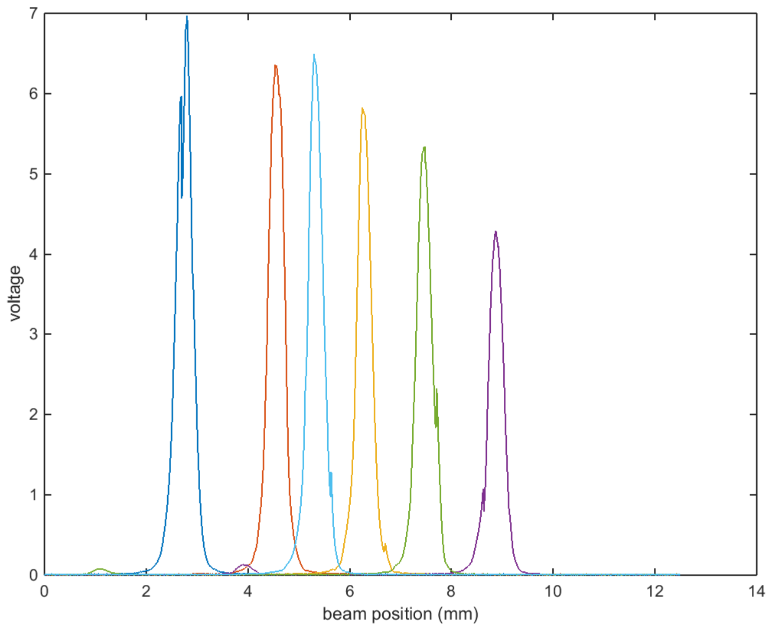

Figure 1: Beam position data measured with the photodiode and the stepper motor. From left, the angles of incidence for the six beam profiles are 0.0°, 30.0°, 40.0°, 50.0°, 60.0°, and 70.0°.

2.3. Beam position measured with the ruler

I measured the beam position with a ruler attached to the screen with binder clips for three angles of incidence. It was somewhat difficult to determine the exact center point of the laser by eye, since the beam spot is small and very bright everywhere. I simply estimate the uncertainty on each measurement to be half the markings on the ruler, or 0.5 mm. Beam displacements (d) are the difference between the measured beam location and the zero position, so the resulting uncertainty is

$$\sigma_d(\text{ruler}) = \sqrt{0.5^2 + 0.5^2} = 0.7 \text{ mm} \quad (2)$$

2.4. Beam position with the photodiode and stepper motor

In the second set of measurements I used the photodiode and stepper motor to measure the beam position. The digitized data is shown in Fig. 1.

I determined the center position of the beam by inspecting the location of the rising and falling edges of the beam profiles with Matlab's zoom tool. I took the mean position between these edges as the beam center. I estimate that this simple procedure results in an uncertainty of 0.01 mm on the beam center for a typical dataset, corresponding to four steps of the stepper motor).

However, the beam profile for the zero degree dataset has an unusual double-peak structure, as seen in Fig. 1. The origin of the double bump is unknown, but it may be the result of multiple

θ_i (deg)	d (mm)	θ_r (deg)	n	$\sigma_n(\theta_i)$	$\sigma_n(L)$	$\sigma_n(d)$
30.00 ± 0.14	2.4 ± 0.7	15 ± 5	1.89 ± 0.86	0.01	0.01	0.86
50.00 ± 0.14	3.1 ± 0.7	34 ± 5	1.38 ± 0.22	0.004	0.004	0.22
70.00 ± 0.14	6.0 ± 0.7	39 ± 10	1.47 ± 0.35	0.007	0.011	0.35

Table 1: Results from the ruler measurements. The thickness of the glass (L) is 9.15 ± 0.05 mm. The uncertainty on n is calculated by varying each of the three inputs (θ_i, L, and d) one at a time and adding the results in quadrature. The uncertainty on n due to each input alone is shown in the last three columns. The weighted average of the three measurements is $n = 1.43 \pm 0.18$.

reflections in the glass block or the photodiode iris. Due to the double bump structure, I estimate that the uncertainty on the beam position for the zero degree dataset is 0.02 mm. After subtracting the zero position, the resulting uncertainty on d is for each measurement is

$$\sigma_d(\text{photodiode}) = \sqrt{0.01^2 + 0.02^2} = 0.022 \text{ mm}. \tag{3}$$

3. Results

The results from the ruler measurements and the photodiode measurements are shown in Tables 1 and 2. The angle of refraction for each dataset was calculated according to

$$\theta_r = \tan^{-1}\left(\tan\theta_i - \frac{|d|}{L\cos\theta_i}\right) \tag{4}$$

The uncertainty on θ_r was calculated with excel by explicitly varying the three input values (θ_i, L, and d) by their uncertainty and observing the resulting variation in θ_r. The three variations were added in quadrature to determine the total uncertainty. Then Snell's law was used to determine the index of refraction for each dataset:

$$n(\text{glass}) = \sin(\theta_\text{i})/\sin(\theta_\text{r}), \tag{5}$$

where again the total uncertainty is determined by varying θ_i, L, and d one at a time. Tables 1 and 2 shows both the total uncertainty on n and the uncertainty due to each of the three input values individually.

The weighted average of the results for the ruler measurements is $n = 1.43 \pm 0.18$, and the weighted average for the photodiode measurements is $n = 1.507 \pm 0.005$

An alternative method to combine the photodiode measurements is shown in Fig. 2, where I have plotted $\sin(\theta_r)$ versus $\sin(\theta_i)$. According to Snell's Law, the slope of these data points are the inverse of the index of refraction:

$$\sin(\theta_r) = \frac{1}{n(\text{glass})}\sin(\theta_i) \tag{6}$$

A linear χ^2 fit with the slope as the only free parameter (y-intercept set to zero) is shown superimposed upon the data in Fig. 2. The fit was performed with the online tool at `http://www.physics.csbsju.edu/stats/WAPP.html`. The uncertainties on $\sin\theta_r$) shown in the figure and included in the χ^2 fit propagate directly from the uncertainties on θ_r as listed in Table 2. The fitted slope is 0.6637 ± 0.0011, which corresponds to $n = (1/\text{slope}) = 1.507 \pm 0.003$. The χ^2 is 1.08 for four degrees of freedom, for a probability of 90%, which is acceptable.

θ_i (deg)	d (mm)	θ_r (deg)	n	$\sigma_n(\theta_i)$	$\sigma_n(L)$	$\sigma_n(d)$
30.00 ± 0.14	1.800 ± 0.022	19.30 ± 0.15	1.513 ± 0.013	0.005	0.005	0.011
40.00 ± 0.14	2.565 ± 0.022	25.32 ± 0.18	1.503 ± 0.011	0.005	0.005	0.008
50.00 ± 0.14	3.523 ± 0.022	30.66 ± 0.21	1.502 ± 0.011	0.005	0.006	0.007
60.00 ± 0.14	4.710 ± 0.022	35.09 ± 0.28	1.507 ± 0.012	0.006	0.008	0.007
70.00 ± 0.14	6.127 ± 0.022	38.30 ± 0.45	1.516 ± 0.017	0.008	0.013	0.009

Table 2: Results from the photodiode + stepper motor data. The thickness of the glass (L) is 9.15 ± 0.05 mm. The uncertainty on n is calculated by varying each of the three inputs (θ_i, L, and d) one at at time and adding the results in quadrature. The uncertainty on n due to each input alone is shown in the last three columns. The weighted average of the five measurements is $n = 1.507 \pm 0.005$

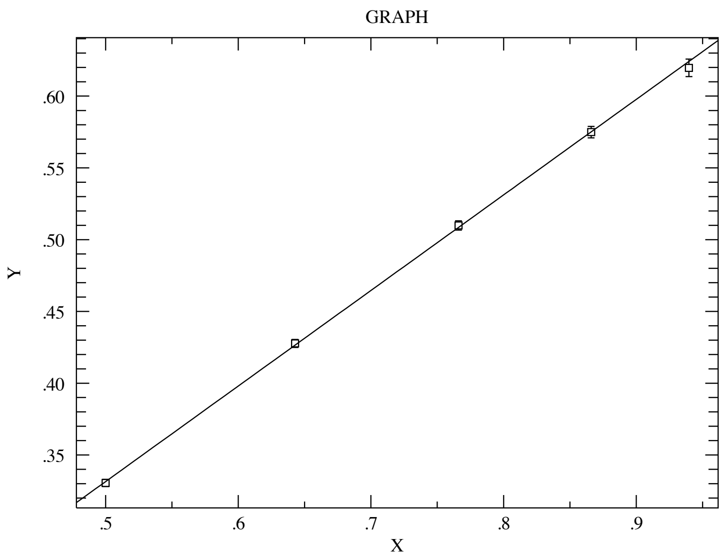

Figure 2: x-axis: $\sin\theta_i$. y-axis: $\sin\theta_r$. A linear χ^2 fit is shown. The χ^2 is 1.08 for four degrees of freedom. *Caveat: The online fitting tool does not draw the fitted line for the one-parameter (slope-only) fit. Instead, I show here the result of a two-parameter fit, where the y-intercept is also allowed to float. The one-parameter linear fit looks similar to this.*

4. Discussion of systematic uncertainties and methods for improvement

For the ruler measurement the uncertainty on the beam position dominates the uncertainty on the index of refraction, as expected. The photodiode and stepper motor provide a much more precise measurement of the beam position, decreasing the uncertainty on the total index of refraction from 0.18 to just 0.005.

For the photodiode measurements, the uncertainty on d is the most important one for the 30, 40, and 50 degree datasets, however the uncertainties on L and θ_i are almost equally important, as seen in the last three columns of Table 2. The measurement of d suffered from a mysterious double-pump beam profile for the zero position dataset, and I adopted a systematic uncertainty on d of 0.02 mm to account for this. The uncertainty on d could be decreased if the double bump could be understood and eliminated. The beam center could also be determined more precisely with a more sophisticated data analysis technique, such as a Gaussian fit to each of the beam profiles near their maxima.

The fractional uncertainty on the glass thickness L could be substantially reduced by using a thicker piece of glass, or by using the thicker dimension of the glass block rather than the thin one. It is worth noting that the uncertainty on L was simply estimated to be the last digit on the calipers reading, because no variation was observed from one measurement to the next.

The measurement of the angle of incidence could be improved with a larger rotation table marked with a finer scale, or by observing the position of the reflected beam on a very fine angular scale set far away from the glass block. Also, more data points could be taken between 40 and 60 degrees incidence, where the uncertainties on the input values propagate to have the smallest impact on n.

Two other possible systematic uncertainties are

1. the assumption that the index of refraction of air is 1.0, and
2. the assumption that the glass block is perfectly rectangular.

Wikipedia indicates that the index of refraction of air is close to 1.0003, which is a very small fractional change compared to our assumption of $n = 1.0$. Therefore this systematic uncertainty appears to be negligible.

If the glass block is not perfectly rectangular, then Equation 4 is not exact. I found with the calipers that the thickness of the glass block does not vary much from place-to-place, consistent with a perfect rectangle. Also, the measured data points for the index of refraction agree well with the Snell's law expectation, as shown in the linear χ^2 fit of Fig. 2. This is consistent with our assumption that Equation 4 applies exactly.

5. Summary

I measured the index of refraction of the glass block to be 1.43 ± 0.18 with a ruler and 1.507 ± 0.005 with the photodiode and stepper motor. The two measurements are in good agreement with each other.